Esercizio

Date le funzioni $f(x)=x^2+1$ e $g(x)=\ln x$ calcolare l'espressione analitica di $h(x)=g\circ f(x)$ e determinare quando $h(x)$ è decrescente

Soluzione

$f(x)=x^2+1$ $\quad D_f: \mathbb{R}$

$g(x)=\ln x$ $\quad D_g:]0,+\infty[$

- $h(x)=g\circ f(x)=g(\underbrace{f(x)}_{x^2+1})=g(\underbrace{x^2+1}_{VAR})=\ln(x^2+1)$

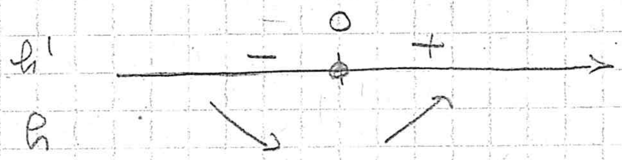
- Calcolo la derivata di $h(x)$ cioè

$$h'(x)=\frac{1}{x^2+1}\cdot 2x=\frac{2x}{x^2+1}$$

- Studio il segno di $h'(x)$ cioè

$h'(x)\geq 0 \quad \frac{2x}{\underbrace{x^2+1}_{\text{sempre }>0}}\geq 0 \Rightarrow \frac{\cancel{2}x}{\cancel{2}}\geq\frac{0}{2} \quad x\geq 0$

Quindi



h è decrescente in $]-\infty;0]$

(h è crescente in $[0,+\infty[$)

(h ha un punto di minimo relativo in $x_0=0$)

Esercizio

Si determinino gli intervalli di crescenza e decrescenza della seguente funzione $f(x)=x\,e^{x}$

Soluzione

$f(x)=x\,e^{x}$ $\qquad D_f: \mathbb{R}$

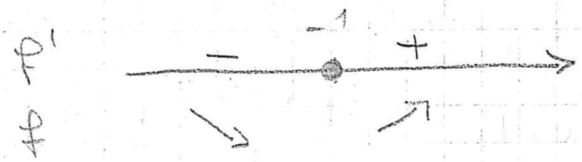
- Calcolo la derivata cioè

$$f'(x)=1\cdot e^{x}+x\,e^{x}=e^{x}(1+x)$$

- Studio il segno di $f'(x)$ cioè

$$f'(x)\geq 0 \qquad \underbrace{e^{x}}_{\text{sempre}>0}(1+x)\geq 0 \Rightarrow 1+x\geq 0 \Rightarrow x\geq -1$$

Quindi



f è decrescente in $]-\infty;-1]$

f è crescente in $[-1;+\infty[$

(f ha un punto di minimo relativo in $x_0=-1$)

Esercizio

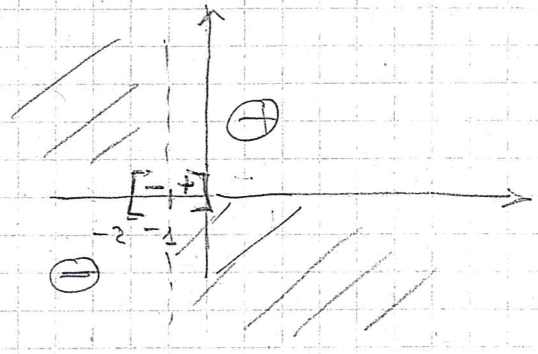
Calcolare l'area della regione di piano compresa tra la funzione $f(x) = x^3 + x^2$, l'asse delle ascisse e le rette $x = -2$ e $x = 0$

Soluzione

$f(x) = x^3 + x^2 = x^2(x+1)$ $\qquad D = \mathbb{R}$

Studio il segno di $f(x)$ cioè

$f(x) \geq 0 \qquad \underbrace{x^2}_{\geq 0}(x+1) \geq 0 \implies x+1 \geq 0 \qquad x \geq -1$



Quindi

$$A = -\int_{-2}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx = (*)$$

$$\int_{-2}^{-1} (x^3 + x^2)\,dx = \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-2}^{-1} = F(-1) - F(-2) =$$

$$= \left[\frac{(-1)^4}{4} + \frac{(-1)^3}{3}\right] - \left[\frac{(-2)^4}{4} + \frac{(-2)^3}{3}\right] =$$

$$= \frac{1}{4} - \frac{1}{3} - \left[\frac{\cancel{16}^{\,4}}{\cancel{4}} - \frac{8}{3}\right] = \frac{1}{4} - \frac{1}{3} - 4 + \frac{8}{3} = \frac{1}{4} - 4 + \frac{7}{3}$$

$$= \frac{3 - 48 + 28}{12} = \frac{-17}{12}$$

$$\int_{-1}^{0} (x^3 + x^2)\,dx = \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{0} = F(0) - F(-1) =$$

$$= 0 - \left[\frac{(-1)^4}{4} + \frac{(-1)^3}{3}\right] = 0 - \left[\frac{1}{4} - \frac{1}{3}\right] = -\frac{1}{4} + \frac{1}{3} = \frac{-3+4}{12} = \frac{1}{12}$$

Quindi

$$(*) = -\left(-\frac{17}{12}\right) + \frac{1}{12} = \frac{17}{12} + \frac{1}{12} = \frac{\cancel{18}^{\,3}}{\cancel{12}_{\,2}} = \frac{3}{2}$$

Calcolare il seguente integrale con il metodo di sostituzione

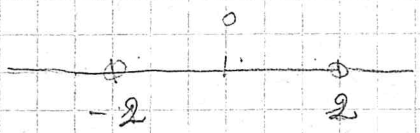
$$\int \frac{4x}{x^2-4}\,dx = 4\int \frac{x}{\underbrace{x^2-4}_{=t}}\,dx \quad \left(x\,dx = \frac{dt}{2}\right)$$

sostituzione

$t = x^2 - 4$

$dt = 2x\,dx$

$\frac{dt}{2} = x\,dx$



$$= 4\int \frac{1}{t}\,\frac{dt}{2} =$$

$$= \frac{\cancel{4}^{\,2}}{\cancel{2}}\int \frac{1}{t}\,dt = 2\ln|t| + c = 2\ln|x^2-4| + c$$

$$\int_{-1}^{0} \frac{4x}{x^2-4} = \left[2\ln|x^2-4|\right]_{-1}^{0} = F(0) - F(-1) =$$

$$= 2\ln|(0)^2-4| - 2\ln|(-1)^2-4| =$$

$$= 2\ln|-4| - 2\ln|-3| =$$

$$= 2\ln 4 - 2\ln 3 = 2(\ln 4 - \ln 3) = 2\ln\left(\frac{4}{3}\right)$$

NB $\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$

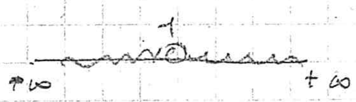
Esercizio

Determinare il dominio e gli eventuali asintoti orizzontali e verticali della funzione:

$$f(x) = 2^{\frac{x^2}{1-x}}$$

Soluzione

$$f(x) = 2^{\frac{x^2}{1-x}} \qquad D_f: \ 1-x \neq 0 \quad -x \neq -1 \quad x \neq 1$$

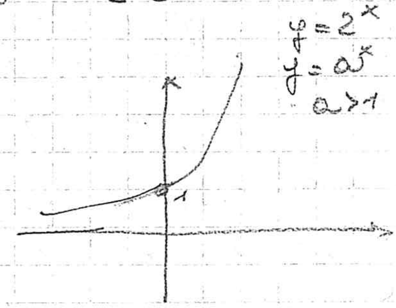


Cerco gli asintoti orizzontali

$$\lim_{x \to +\infty} 2^{\frac{x^2}{1-x}} = \lim_{x \to +\infty} 2^{\frac{x^2}{-x}} = \lim_{x \to +\infty} 2^{-x} = 2^{-\infty} = 0$$

quindi $y=0$ è asintoto orizzontale a dx

NB



$$\lim_{x \to -\infty} 2^{\frac{x^2}{1-x}} = \lim_{x \to -\infty} 2^{\frac{x^2}{-x}} = \lim_{x \to -\infty} 2^{-x}$$

$$= 2^{-(-\infty)} = 2^{+\infty} = +\infty$$

qui non c'è asintoto orizzontale a sx

Cerco gli asintoti verticali

$$\lim_{x \to 1^+} 2^{\frac{x^2}{1-x}} = 2^{\frac{1}{1-1}} = 2^{\frac{1}{0^-}} = 2^{-\infty} = 0$$

qui non c'è asintoto verticale

$1,1 \qquad 1-1,1 = -0,1 = 0^-$

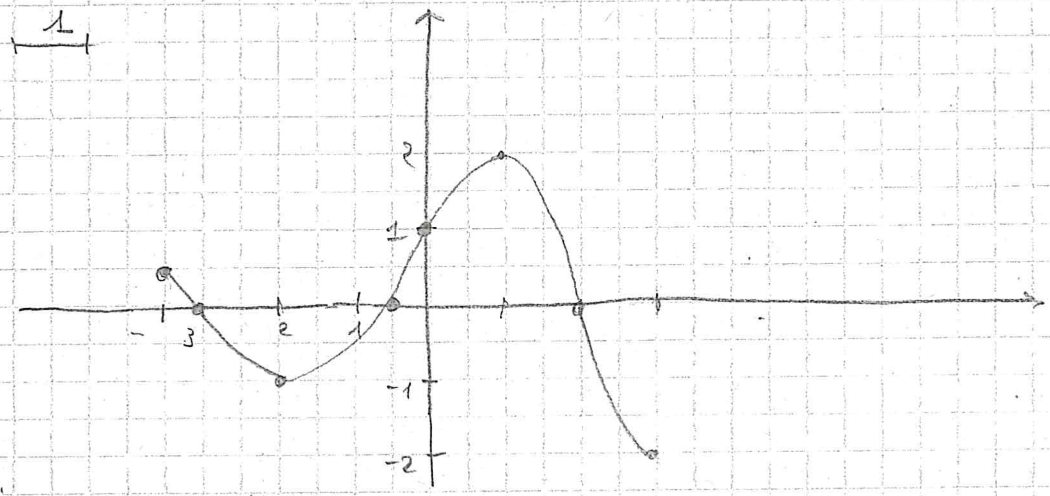
$$\lim_{x \to 1^-} 2^{\frac{x^2}{1-x}} = 2^{\frac{1}{1-1}} = 2^{\frac{1}{0^+}} = 2^{+\infty} = +\infty$$

quindi $x=1$ è asintoto verticale

$0,9 \qquad 1-0,9 = 0,1 = 0^+$

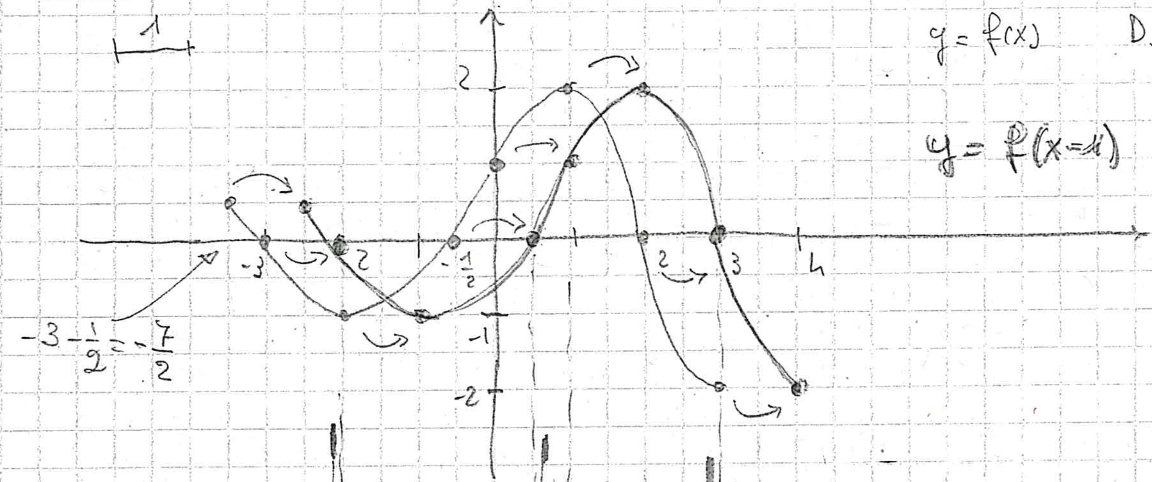
Esercizio

Partendo dal grafico di $f(x)$:



Rappresentare graficamente $g(x) = \dfrac{1}{f(x-1)}$ e indicarne il dominio

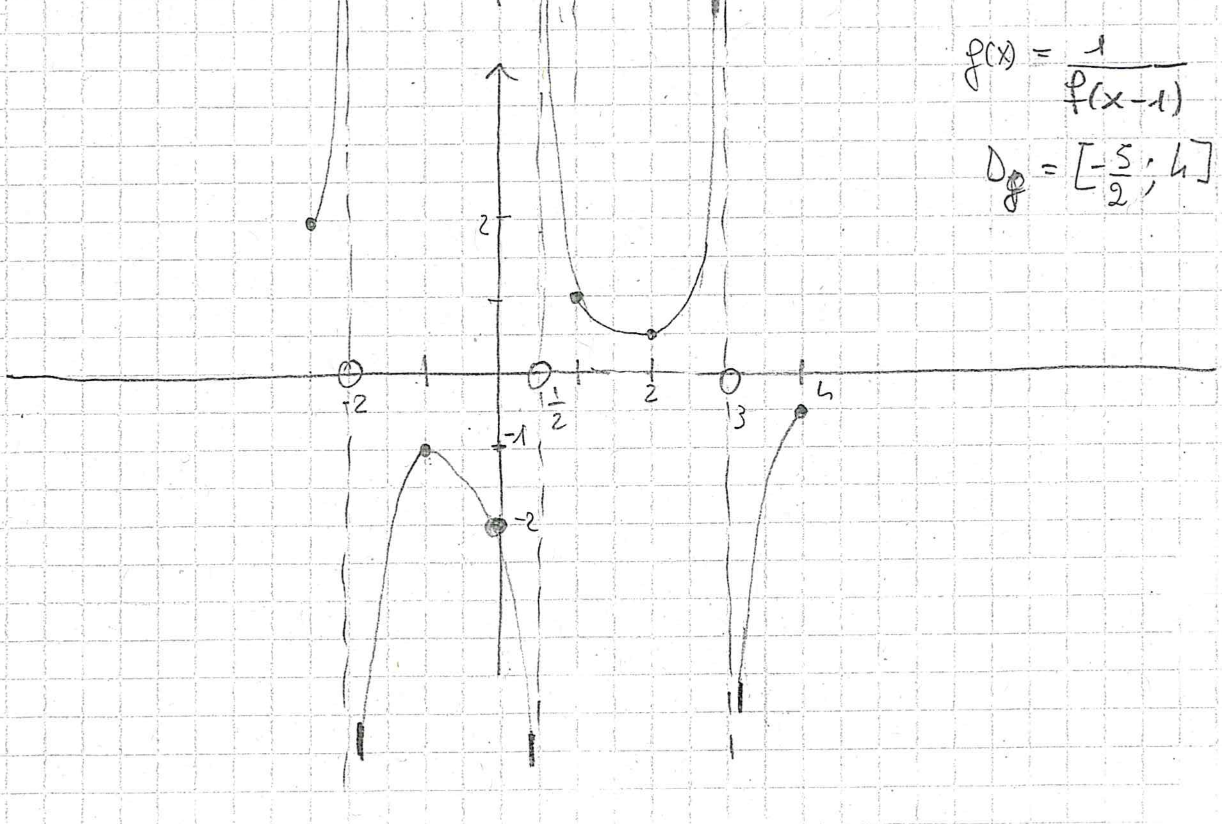
Soluzione



$y = f(x)$ $\quad D_f = \left[-\frac{7}{2}; 3\right]$ $\quad \left(-3-\frac{1}{2}\right)$

$y = f(x-1)$ $\quad D: \left[-\frac{5}{2}; 4\right]$

$-\frac{7}{2}+1 = -\frac{5}{2}$



$$g(x) = \frac{1}{f(x-1)}$$

$$D_g = \left[-\frac{5}{2}; 4\right]$$

Esercizio

Calcolare, se possibile, la matrice inversa di $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

Soluzione

$$\det(A) = \left[\begin{array}{ccc|cc} 0 & 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & -1 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] = 0+1+0+1+0+0 = 2 \neq 0 \Rightarrow$$

$\Rightarrow$ A è una matrice invertibile ed ammette la matrice inversa A^{-1}

NB: $\boxed{\text{Il pivot deve essere } \neq 0}$

Calcolo la matrice inversa

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \sim \begin{array}{c} R_1+R_3 \\ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\begin{array}{c} \\ \\ R_3-R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 \end{array}\right] \sim \begin{array}{c} \\ \\ R_3-R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 0 \end{array}\right] \sim$$

$$\begin{array}{c} \\ -R_2 \\ -\frac{1}{2}R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right] \sim \begin{array}{c} R_1-3R_3 \\ R_2+R_3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -\frac{1}{2} & -\frac{3}{2} & 1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right] \sim$$

$$\begin{array}{c} R_1-2R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{2} & -\frac{1}{2} & 1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right]$$

Quindi la matrice inversa è

$$A^{-1} = \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Esercizio

Risolvere $\log_3(x^2+1) > \log_3(2x+4)$

Ricordare che:

$$\log_a f(x) \gtrless \log_a g(x) \Rightarrow f(x) \gtrless g(x) \quad \text{se } a>1 \quad \begin{pmatrix}\text{MANTENGO} \\ \text{IL VERSO}\end{pmatrix}$$

$$\Rightarrow f(x) \lessgtr g(x) \quad \text{se } 0<a<1 \quad \begin{pmatrix}\text{CAMBIO} \\ \text{IL VERSO}\end{pmatrix}$$

Soluzione

$\log_3(x^2+1) > \log_3(2x+4)$

base $a=3>1 \Rightarrow$ MANTENGO IL VERSO

$x^2+1 > 2x+4$

$x^2-2x-3>0$ paraboloU

Dominio

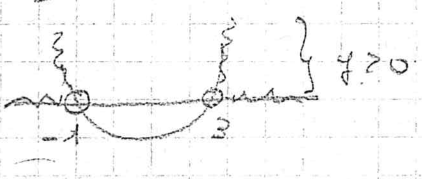
$$\begin{cases} x^2+1>0 & \text{sempre } \forall x \\ 2x+4>0 & \frac{\cancel{2}x}{\cancel{2}} > -\frac{4}{\cancel{2}}^{2} \end{cases}$$

$$\begin{cases} \forall x \\ x>-2 \end{cases}$$

$x^2-2x-3=0$

$$x_{1,2} = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-3)}}{2(1)} =$$

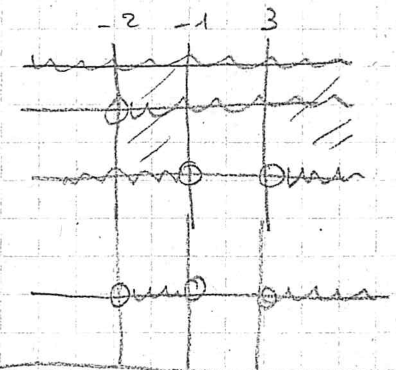
$$= \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \begin{array}{l} -1 \\ 3 \end{array}$$



esterni:

$x<-1 \vee x>3$

$$\begin{cases} \forall x \\ x>-2 \\ x<-1 \vee x>3 \end{cases}$$



Soluzione

$\boxed{-2<x<-1 \vee x>3}$

OK