INTEGRALI INDEFINITI

$$\int f(x)\,dx = \Big\{ \overbrace{F(x)+c}^{\text{PRIMITIVA di } f} \;\Big|\; F'(x) = f(x) \Big\}$$

quindi $\int \underbrace{f(x)}_{\overset{\shortparallel}{F'(x)}}\,dx = F(x)+c$ cioè $f(x) = (F(x)+c)'$

- $\int 1\,dx = x+c$ $\qquad \int 0\cdot dx = c$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

- $\int x\,dx = \frac{x^2}{2}+c$
- $\int x^2 dx = \frac{x^3}{3}+c$
- $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}dx = \frac{x^{\frac{1}{2}+1 = \frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3}+c$
- $\int \sqrt[3]{x^2}\,dx = \int x^{\frac{2}{3}}dx = \frac{x^{\frac{2}{3}+1 = \frac{5}{3}}}{\frac{5}{3}} + c = \frac{3}{5}\sqrt[3]{x^5}+c$
- $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}dx = \frac{x^{-\frac{1}{2}+1 = \frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{x}+c$
- $\int \frac{1}{x^2}\,dx = \int x^{-2}dx = \frac{x^{-1}}{-1}+c = -\frac{1}{x}+c$
- $\int \frac{1}{x^3}\,dx = \int x^{-3}dx = \frac{x^{-2}}{-2}+c = -\frac{1}{2x^2}+c$ $\qquad \int \frac{1}{x}\,dx = \ln|x|+c$
- $\int \underset{\text{deriv.}}{\frac{1}{x}}\,dx = \int x^{-1}dx = \frac{x^{-1+1}}{-1+1}+c = \frac{x^0}{\cancel{0}}+c$ non funziona

$= \ln|x|+c$ notare il valore assoluto per essere coerente con il dominio di $f(x) = \frac{1}{x}$ che è $x \neq 0$

- $\int \underbrace{\cos x}_{\text{deriv.}}\, dx = \operatorname{sen} x + c$

- $\int \operatorname{sen} x\, dx = -\int \underbrace{-\operatorname{sen} x}_{\text{deriv.}}\, dx = -\cos x + c$

- $\int e^x\, dx = e^x + c$

$\int k\, f(x)\, dx = k \int f(x)\, dx$

$\int f(x) \pm g(x) = \int f(x) + \int g(x)$

- $\int\left(\frac{x^2}{3} - \frac{x}{2} + 4x^3 - 2\right) dx = \frac{1}{3}\int x^2 dx - \frac{1}{2}\int x\, dx + 4\int x^3 dx - 2\int dx =$

$= \frac{1}{3}\frac{x^3}{3} - \frac{1}{2}\cdot\frac{x^2}{2} + 4\cdot\frac{x^4}{4} - 2x + c = \frac{x^3}{9} - \frac{x^2}{4} + x^4 - 2x + c$

$\int\left(\frac{1}{\sqrt[3]{x^2}} - \frac{5}{x^3} + \sqrt[5]{x}\right) dx = \int x^{-\frac{2}{3}} dx - 5\int x^{-3} dx + \int x^{\frac{1}{5}} dx =$

$= \frac{x^{\left(-\frac{2}{3}+1\right) = \frac{1}{3}}}{\frac{1}{3}} - 5\frac{x^{-2}}{-2} + \frac{x^{\left(\frac{1}{5}+1\right) = \frac{6}{5}}}{\frac{6}{5}} + c = 3\sqrt[3]{x} + \frac{5}{2x^2} + \frac{5}{6}\sqrt[5]{x^6} + c$

$\int \frac{2 + x^2 - x^4}{2}\, dx = \int\left(\frac{\cancel{2}^1}{\cancel{2}} + \frac{x^2}{2} - \frac{x^4}{2}\right) dx = \int dx + \frac{1}{2}\int x^2 dx - \frac{1}{2}\int x^4 dx =$

$= x + \frac{x^3}{3} - \frac{1}{2}\cdot\frac{x^5}{5} + c$

$\int \frac{x^3 - 4x^2 + x + 1}{x}\, dx = \int\left(\frac{x^{\cancel{3}\,2}}{\cancel{x}} - \frac{4x^{\cancel{2}}}{\cancel{x}} + \frac{\cancel{x}^1}{\cancel{x}} + \frac{1}{x}\right) dx = \int x^2 dx - 4\int x\, dx + \int dx + \int \frac{1}{x} dx$

$= \frac{x^3}{3} - \frac{\cancel{4}^2 x^2}{\cancel{2}} + x + \ln|x| + c$

$\int \frac{\sqrt{x} + 2x + 1}{x^5} = \int\left(\frac{\sqrt{x}}{x^5} + \frac{2\cancel{x}}{x^{\cancel{5}4}} + \frac{1}{x^5}\right) dx = \int\left(x^{\left(\frac{1}{2}-5\right) = \frac{-9}{2}} + 2x^{-4} + x^{-5}\right) dx =$

$= \frac{x^{\left(-\frac{9}{2}+1\right) = -\frac{7}{2}}}{-\frac{7}{2}} + 2\frac{x^{-3}}{-3} + \frac{x^{-4}}{-4} + c = -\frac{2}{7\cdot\sqrt{x^7}} - \frac{2}{3\cdot x^3} - \frac{1}{4x^4} + c$

②

$$\int (2\cos x - 3\,\text{sen}\,x + 5)\,dx = 2\,\text{sen}\,x - 3(-\cos x) + 5x + c$$
$$= 2\,\text{sen}\,x + 3\cos x + 5x + c$$

$$\int \frac{\text{sen}\,x - 5\cos x + 3x}{4}\,dx = \int \left(\frac{\text{sen}\,x}{4} - \frac{5\cos x}{4} + \frac{3x}{4}\right) dx$$
$$= -\frac{1}{4}\cos x - \frac{5}{4}\,\text{sen}\,x + \frac{3}{4}\cdot\frac{x^2}{2} + c$$
$$= -\frac{1}{4}\cos x - \frac{5}{4}\,\text{sen}\,x + \frac{3}{12}x^2 + c$$

$$\int \left(e^x + 4x + \frac{1}{x}\right) dx = e^x + \frac{\cancel{4}^{2} x^2}{\cancel{2}} + \ln|x| + c =$$
$$= e^x + 2x^2 + \ln|x| + c$$

$$\int \frac{1}{1+x^2}\,dx = \text{arctg}\,x + c$$

$$\int \frac{-1}{1+x^2}\,dx = \text{arccotg}\,x + c$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \text{arcsen}\,x + c$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + c$$