

Calcolo di limiti per $x \rightarrow \pm\infty$ con funzioni basate su polinomi

$$\lim_{x \rightarrow +\infty} -x^2 - 5x^7 + 1 = \lim_{x \rightarrow +\infty} -5x^7 = -5(+\infty)^7 = -5(+\infty) = -\infty$$

$$\lim_{x \rightarrow -\infty} -x^2 - 5x^7 + 1 = \lim_{x \rightarrow -\infty} -5x^7 = -5(-\infty)^7 = -5(-\infty) = +\infty$$

$$\lim_{x \rightarrow -\infty} -x^2 - 5x^4 + 1 = \lim_{x \rightarrow -\infty} -5x^4 = -5(+\infty)^4 = -5(+\infty) = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{-4x^8 + x - 3}{x^2 - 3x^5 + 1} = \lim_{x \rightarrow -\infty} \frac{-4x^8}{-3x^5} = \frac{-4(-\infty)^3}{-3} = \frac{-4(-\infty)}{-3} = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x - 4x^{10} - 3}{-x^2 + x} = \lim_{x \rightarrow -\infty} \frac{-4x^{10}}{-x^2} = \frac{-4(-\infty)^8}{-1} = \frac{-4(+\infty)}{-1} = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x - 4x^{10} - 3}{-x^{10} + x} = \lim_{x \rightarrow -\infty} \frac{-4x^{10}}{-x^{10}} = \frac{-4}{-1} = 4$$

$$\lim_{x \rightarrow +\infty} \frac{x + 3x^8 - 3}{-6x^8 + 4} = \lim_{x \rightarrow +\infty} \frac{3x^8}{-6x^8} = \frac{3}{-6} = -\frac{1}{2}$$

$$\lim_{x \rightarrow -\infty} \frac{x - x^3 - 3}{1 - 2x^4} = \lim_{x \rightarrow -\infty} \frac{-x^3}{-2x^4} = \frac{-1}{-2(-\infty)} = \frac{-1}{+\infty} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{x - x^3 - 3}{1 - 2x^5} = \lim_{x \rightarrow -\infty} \frac{-x^3}{-2x^5} = \frac{-1}{-2(-\infty)^2} = \frac{-1}{-\infty} = 0$$

Calcolo dei limiti per $x \rightarrow x_0$ con funzioni basate su polinomi

1) $\lim_{x \rightarrow 1^+} \frac{x}{x-1} = \frac{1}{0^+} = +\infty$

5) $\lim_{x \rightarrow 0^+} \frac{1}{x^2} = \frac{1}{0^+} = +\infty$

2) $\lim_{x \rightarrow 1^-} \frac{x}{1-x} = \frac{1}{0^+} = +\infty$

6) $\lim_{x \rightarrow 1^\pm} \frac{1}{(x-1)^2} = \frac{1}{0^+} = +\infty$

3) $\lim_{x \rightarrow (-1)^+} \frac{x}{x+1} = \frac{1}{0^+} = +\infty$

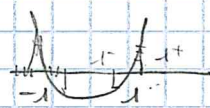
7) $\lim_{x \rightarrow 1^\pm} \frac{-1}{(1-x)^2} = \frac{-1}{0^+} = (-1) \cdot \frac{1}{0^+} = (-1)(+\infty) = -\infty$

4) $\lim_{x \rightarrow (-1)^-} \frac{x}{x+1} = \frac{1}{0^-} = -\infty$

8) $\lim_{x \rightarrow (-1)^\pm} \frac{-1}{(x+1)^2} = \frac{-1}{0^+} = (-1) \cdot \frac{1}{0^+} = (-1)(+\infty) = -\infty$

9) $\lim_{x \rightarrow 1^+} \frac{-1}{x^2-1} = \frac{-1}{0^+} = (-1) \cdot \left(\frac{1}{0^+}\right) = (-1)(+\infty) = -\infty$
 $1,1 \rightarrow (1,1)^2 - 1 = \rightarrow$

$y = x^2 - 1$

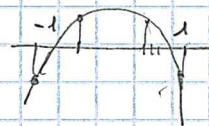


10) $\lim_{x \rightarrow 1^-} \frac{-1}{x^2-1} = \frac{-1}{0^-} = (-1) \cdot \left(\frac{1}{0^-}\right) = (-1)(-\infty) = +\infty$
 $0,9 \rightarrow (0,9)^2 - 1 = \rightarrow$

11) $\lim_{x \rightarrow -1^+} \frac{-1}{x^2-1} = \frac{-1}{0^-} = (-1) \cdot \left(\frac{1}{0^-}\right) = (-1)(-\infty) = +\infty$
 $-0,9 \rightarrow (-0,9)^2 - 1 = \rightarrow$

12) $\lim_{x \rightarrow -1^-} \frac{-1}{x^2-1} = \frac{-1}{0^+} = (-1) \cdot \left(\frac{1}{0^+}\right) = (-1)(+\infty) = -\infty$
 $-1,1 \rightarrow (-1,1)^2 - 1 = \rightarrow$

$y = -x^2 + 1$



13) $\lim_{x \rightarrow 1^+} \frac{-x}{1-x^2} = \frac{-1}{0^-} = (-1) \cdot \left(\frac{1}{0^-}\right) = (-1)(-\infty) = +\infty$
 $1,1 \rightarrow 1 - (1,1)^2 = \rightarrow$

14) $\lim_{x \rightarrow 1^-} \frac{-x}{1-x^2} = \frac{-1}{0^+} = (-1) \cdot \left(\frac{1}{0^+}\right) = (-1)(+\infty) = -\infty$
 $0,9 \rightarrow 1 - (0,9)^2 = \rightarrow$

15) $\lim_{x \rightarrow -1^+} \frac{+x}{1-x^2} = \frac{+1}{0^+} = +\infty$
 $-0,9 \rightarrow 1 - (-0,9)^2 = \rightarrow$

16) $\lim_{x \rightarrow -1^-} \frac{-x}{1-x^2} = \frac{1}{0^-} = -\infty$
 $-1,1 \rightarrow 1 - (-1,1)^2 = \rightarrow$

$$17) \lim_{x \rightarrow 1^+} \frac{2x}{x^2+2x-3} = \frac{2(1)}{0^+} = 2 \cdot \left(\frac{1}{0^+}\right) = 2 \cdot (+\infty) = +\infty$$

$$18) \lim_{x \rightarrow 1^-} \frac{2x}{x^2+2x-3} = \frac{2(1)}{0^-} = 2 \cdot \left(\frac{1}{0^-}\right) = 2 \cdot (-\infty) = -\infty$$

$$19) \lim_{x \rightarrow (-3)^+} \frac{2x}{x^2+2x-3} = \frac{2(-3)}{0^-} = \frac{-6}{0^-} = -6 \cdot \left(\frac{1}{0^-}\right) = -6 \cdot (-\infty) = +\infty$$

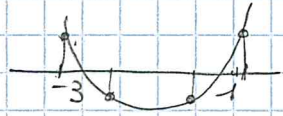
$$20) \lim_{x \rightarrow (-3)^-} \frac{2x}{x^2+2x-3} = \frac{2(-3)}{0^+} = \frac{-6}{0^+} = -6 \cdot \left(\frac{1}{0^+}\right) = -6 \cdot (+\infty) = -\infty$$

$$y = x^2 + 2x - 3$$

$$x^2 + 2x - 3 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4 + 4(1)(-3)}}{2}$$

$$= \frac{-2 \pm 4}{2} \quad \begin{matrix} -3 \\ -1 \end{matrix}$$



$$21) \lim_{x \rightarrow 0^-} \frac{-x^2 + x - 4}{x^4 - 2x} = \frac{-0 + 0 - 4}{0^+} = \frac{-4}{0^+} = -4 \cdot \left(\frac{1}{0^+}\right) = -4 \cdot (+\infty) = -\infty$$

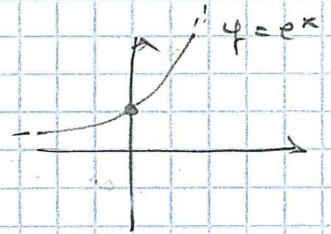
$$22) \lim_{x \rightarrow 0^-} \frac{x^2 + x - 4}{-x^4 + 2x} = \frac{0 + 0 - 4}{0^-} = \frac{-4}{0^-} = (-4) \cdot \left(\frac{1}{0^-}\right) = (-4) \cdot (-\infty) = +\infty$$

$$23) \lim_{x \rightarrow 1^-} \frac{x-4}{x^2(x-1)} = \frac{1-4}{(+)(0^-)} = \frac{-3}{0^-} = -3 \cdot \left(\frac{1}{0^-}\right) = -3 \cdot (-\infty) = +\infty$$

$$24) \lim_{x \rightarrow -1^-} \frac{x-4}{x(x+1)} = \frac{-1-4}{(-)(0^-)} = \frac{-5}{0^+} = -5 \cdot \left(\frac{1}{0^+}\right) = -5 \cdot (+\infty) = -\infty$$

Calcolo dei limiti con funzioni esponenziali.

$$\lim_{x \rightarrow 0} (x^2 - 1) e^x = (0 - 1) e^0 = (-1)(1) = -1$$



$$\lim_{x \rightarrow 0^{\pm}} \frac{x^2 - 1}{e^x} = \frac{0 - 1}{1} = \frac{-1}{1} = -1$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 1}{e^{x-2}} = \frac{4 - 1}{e^0} = \frac{3}{1} = 3$$

$$\lim_{x \rightarrow 0} \frac{x^2 - 1}{e^x - 2} = \frac{0 - 1}{e^0 - 2} = \frac{-1}{-1} = 1$$

$$\lim_{x \rightarrow +\infty} (x^2 - 1) e^x = (+\infty) \cdot e^{+\infty} = (+\infty) \cdot (+\infty) = +\infty$$

$$\lim_{x \rightarrow +\infty} (1 - x^2) e^x = (-\infty) \cdot e^{+\infty} = (-\infty) \cdot (+\infty) = -\infty$$

$$\lim_{x \rightarrow +\infty} \frac{1 - x^2}{e^x} = \lim_{x \rightarrow +\infty} \frac{-1}{e^x} = \left[\frac{-1}{+\infty} \right] = \left[\frac{-1}{+\infty} \right] = 0^-$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{1 - x^2} = \lim_{x \rightarrow +\infty} \frac{e^x}{-x^2} = \lim_{x \rightarrow +\infty} \frac{e^x}{-1} = \left[\frac{+\infty}{-1} \right] = -\infty$$

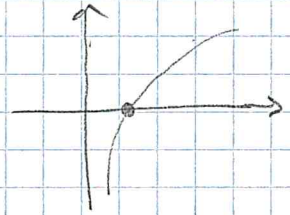
$$\lim_{x \rightarrow -\infty} \frac{e^x}{1 - x^2} = \lim_{x \rightarrow -\infty} \frac{e^x}{-x^2} = \left[\frac{0^+}{-\infty} \right] = \left[0^+ \cdot \left(\frac{1}{-\infty} \right) \right] = 0^+ \cdot (0^-) = 0^-$$

Calcolo dei limiti con le funzioni logaritmiche

$$\lim_{x \rightarrow 1} \sqrt{x + \ln x} = \sqrt{1 + \underbrace{\ln 1}_0} = \sqrt{1+0} = 1$$

$$\lim_{x \rightarrow 4} \frac{\log_2 x + 1}{3 \log_2 x} = \frac{\log_2 4 + 1}{3 \log_2 4} = \frac{2+1}{3 \cdot 1} = \frac{3}{3} = 1$$

$$\lim_{x \rightarrow 1^+} \frac{x}{\ln x} = \frac{1}{\ln 1^+} = \frac{1}{0^+} = +\infty$$



$$\lim_{x \rightarrow 1^-} \frac{x}{\ln x} = \frac{1}{\ln 1^-} = \frac{1}{0^-} = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{x} = \frac{\ln 0^+}{0^+} = \frac{-\infty}{0^+} = -\infty \cdot \left(\frac{1}{0^+}\right) = (-\infty) \cdot (+\infty) = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{x}{\ln x} = \frac{0^+}{\ln 0^+} = \frac{0^+}{-\infty} = 0^+ \cdot \left(\frac{1}{-\infty}\right) = 0^+ \cdot 0^- = 0^-$$

$$\lim_{x \rightarrow +\infty} \frac{\ln x}{x} = \lim_{x \rightarrow +\infty} \frac{1}{x} = \left[\frac{1}{+\infty} \right] = 0$$

$$\lim_{x \rightarrow +\infty} \frac{x}{\ln x} = \lim_{x \rightarrow +\infty} x = +\infty$$