"Regole applicative" per determinare il dominio di una funzione

1) Se $f(x) = \dfrac{N(x)}{D(x)}$ $\Longrightarrow$ DOMINIO: $D(x) \neq 0$

[cioè denominatore $\neq 0$]

2) Se $f(x) = \sqrt[\overset{(PARI)}{n}]{R(x)}$ con n indice pari $\Longrightarrow$ DOMINIO: $R(x) \geq 0$

[cioè radicando ≥ 0]

3) Se $f(x) = \log_a A(x)$ con la base $a > 0$ e $a \neq 1$

$\Longrightarrow$ DOMINIO: $A(x) > 0$

$\cancel{\geq 0}$

[cioè argomento ≥ 0]

4) Altrimenti DOMINIO $= \mathbb{R}$

Trovare il dominio delle seguenti funzioni

$f(x) = x^2 - 1 \qquad D_f = \mathbb{R}$

$f(x) = \dfrac{x-1}{2} \qquad D_f : \mathbb{R}$

$f(x) = \dfrac{1}{2x} \qquad D_f : 2x \neq 0 \Rightarrow x \neq 0$

$D_f : \mathbb{R} \setminus \{0\}$

$f(x) = \dfrac{1}{2x^2} \qquad D_f : 2x^2 \neq 0 \Rightarrow x \neq 0$

$D_f : \mathbb{R} \setminus \{0\}$

$f(x) = \sqrt[4]{1-x} \qquad D_f : 1-x \geq 0 \Rightarrow -x \geq -1 \Rightarrow x \leq 1$

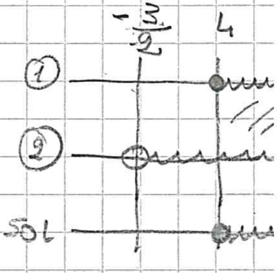
$D_f : \,]-\infty ; 1]$

$f(x) = \dfrac{1}{\sqrt[4]{1-x}} \qquad D_f : 1-x > 0 \Rightarrow -x > -1 \Rightarrow x < 1$

$\cancel{\geq 0}$ $\qquad D_f : \,]-\infty ; 1[$

(*) $f(x) = \dfrac{\sqrt[4]{x-4}}{\sqrt[4]{2x+3}} \qquad D_f : \begin{cases} x-4 \geq 0 \\ 2x+3 > 0 \end{cases} \quad \begin{cases} x \geq 4 \\ x > -\frac{3}{2} \end{cases}$

$\cancel{\geq 0}$



$D_f : x \geq 4$

$[4 ; +\infty[$

(**) $g(x) = \sqrt[4]{\dfrac{x-4}{2x+3}} \qquad D_g : \begin{cases} \dfrac{x-4}{2x+3} \geq 0 \\ 2x+3 \neq 0 \end{cases} \Rightarrow \dfrac{x-4}{2x+3} \geq 0$

$N(x) \geq 0 \quad x-4 \geq 0 \quad x \geq 4$

$D(x) > 0 \quad 2x+3 > 0 \quad x > -\frac{3}{2}$

		$-\frac{3}{2}$		4	
$N(x)$	$-$		$-$		$+$
$D(x)$	$-$		$+$		$+$
$\frac{N(x)}{D(x)}$	$+$	$\nexists$	$-$		$+$
Sol					

$D_g : x < -\frac{3}{2} \vee x \geq 4$

NB. $f(x) = \dfrac{\sqrt[4]{x-4}}{\sqrt[4]{2x+3}}$ e $g(x) = \sqrt[4]{\dfrac{x-4}{2x+3}}$ $\Rightarrow$ $f(x)$ e $g(x)$ sono funzioni diverse e in generale $D_f \neq D_g$

$[4 ; +\infty[\; \neq \;]-\infty ; -\frac{3}{2}[\, \cup \, [4 ; +\infty[$

NOTARE BENE

$$f(x) = \frac{\sqrt[6]{2x}}{\sqrt[6]{x^2-4}} \qquad D_f: \begin{cases} 2x \geq 0 & (1) \\ x^2-4 > 0 & (2) \end{cases}$$

(1) $\frac{\cancel{2}x}{\cancel{2}} \geq \frac{0}{2}$ $\quad x \geq 0$

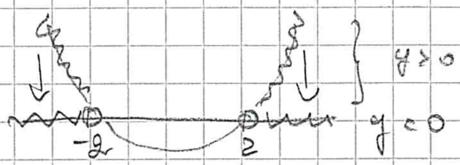
(2) $x^2-4>0$ PARABOLA $\cup$ $y>0$

$I_x: x^2-4=0$

$x^2=4$

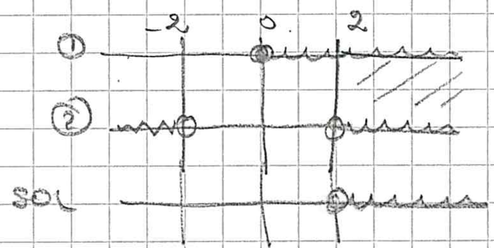
$x=\pm\sqrt{4}$

$x=\pm 2$



$x<-2 \ \vee \ x>2$

$$\begin{cases} x \geq 0 \\ x<-2 \ \vee \ x>2 \end{cases}$$



$D_f: \ x>2$

$]2; +\infty[$

$$g(x) = \sqrt[6]{\frac{2x}{x^2-4}} \qquad D_g: \begin{cases} \frac{2x}{x^2-4} \geq 0 \\ x^2-4 \neq 0 \end{cases}$$

$\frac{2x}{x^2-4} \geq 0$

$N(x) \geq 0 \quad \frac{\cancel{2}x}{\cancel{2}} \geq \frac{0}{2} \quad x \geq 0$

$D(x)>0 \quad x^2-4>0$ PARABOLA $\cup$ $y>0$

(stessi passaggi dell'es. precedente)

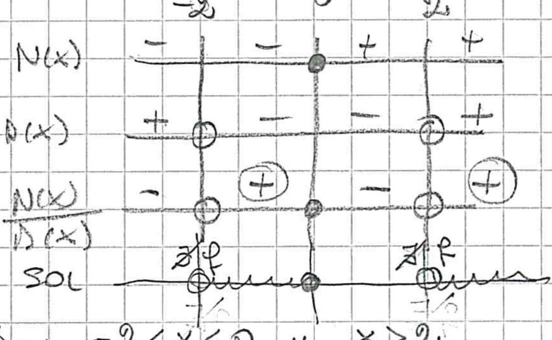
$x<-2 \ \vee \ x>2$

N.B.

$f(x)$ e $g(x)$ sono funzioni diverse e in generale hanno domini diversi

$D_f: \ ]2; +\infty[$

$D_g: \ ]-2; 0] \ \cup \ ]2; +\infty[$



$D_g: \ -2<x\leq 0 \ \vee \ x>2$

$]-2; 0] \ \cup \ ]2; +\infty[$