

# EQUAZIONI DI 1° GRADO

$$1) 7x - 3 + 5(-2x + 1) = 3x - 7$$

$$7x - 3 - 10x + 5 = 3x - 7$$

$$7x - 10x - 3x = 3 - 7$$

$$\frac{-6x}{-6} = \frac{-4}{-6}$$

2° principio equazioni

$$x = + \frac{4}{6} \cdot 2$$

$$x = \frac{2}{3}$$

$$2) -\frac{1}{2} - 5(-x - 1) = 2 \left( \frac{13}{5} + x \right) + \frac{1}{2}$$

$$-\frac{1}{2} + 5x + 5 = \frac{26}{5} + 2x + \frac{1}{2} \quad (\text{no trasposto})$$

$$10 \cdot \frac{-5 + 50x + 50}{10} = \frac{52 + 20x + 5}{10} \cdot 10 \quad 2^\circ \text{ principio eq.}$$

$$50x - 20x = 5 - 50 + 52 + 5$$

$$\frac{30x}{30} = \frac{12}{30}$$

2° principio eq.

$$x = \frac{12}{30} \cdot \frac{1}{5}$$

$$x = \frac{2}{5}$$

$$3) \frac{3x + 5}{2} - \frac{x - 1}{14} = \frac{8x - 5}{7}$$

$$14 \cdot \frac{7(3x + 5) - 1(x - 1)}{14} = \frac{2(8x - 5)}{14} \cdot 14 \quad 2^\circ \text{ principio eq.}$$

$$21x + 35 - x + 1 = 16x - 10$$

$$21x - x - 16x = -35 - 1 - 10$$

$$\frac{4x}{4} = \frac{-46}{4}$$

2° principio eq.

$$x = \frac{-46}{4} \cdot \frac{1}{2}$$

$$x = -\frac{23}{2}$$



# EQUAZIONI DI 2°

$$ax^2 + bx + c = 0 \quad \text{con } a \neq 0$$

COMPLETE

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\overbrace{a}^2 x^2 + \overbrace{b}^{-3} x + \overbrace{c}^{-4} = 0$$

$$x_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)} = \frac{+3 \pm \sqrt{9+16}}{2}$$
$$= \frac{3 \pm 5}{2} \quad \left\{ \begin{array}{l} \frac{-2}{2} = -1 \\ \frac{8}{2} = 4 \end{array} \right.$$

2 soluzioni distinte

$$\overbrace{a}^9 x^2 + \overbrace{b}^6 x + \overbrace{c}^1 = 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{(6)^2 - 4(9)(1)}}{2(9)}$$
$$= \frac{-6 \pm \sqrt{36 - 36}}{18} = \frac{-6 \pm \sqrt{0}}{18} = \frac{-6}{18}$$
$$= -\frac{1}{3} \quad \left\{ \begin{array}{l} x_1 = -\frac{1}{3} \\ x_2 = -\frac{1}{3} \end{array} \right.$$

2 soluzioni coincidenti

$$\overbrace{a}^{-1} x^2 + \overbrace{b}^1 x + \overbrace{c}^{-1} = 0$$

$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)(-1)}}{2(-1)} = \frac{-1 \pm \sqrt{1-4}}{-2}$$
$$= \frac{-1 \pm \sqrt{-3}}{-2}$$

impossibile  
nessuna soluzione

## INCOMPLETE

I)  $x^2 - 2x = 0$  RACCOLGO  $x \Rightarrow x(x-2) = 0 \Rightarrow x_1 = 0 \vee x_2 = 2$

II)  $2x^2 - 6 = 0$  RICAVO  $x^2 \Rightarrow \frac{2}{2}x^2 = \frac{6}{2}$   
 $x^2 = 3 \quad x = \pm\sqrt{3}$   
2 soluz. distinte

$2x^2 + 6 = 0$   $\Rightarrow \frac{2}{2}x^2 = \frac{-6}{2}$   
 $x^2 = -3 \quad x = \pm\sqrt{-3}$   
impossibile


$3x^2 = 0$   $\Rightarrow \frac{3}{3}x^2 = \frac{0}{3}$   
 $x^2 = 0 \quad x_{1,2} = \pm\sqrt{0}$   
 $x = 0$

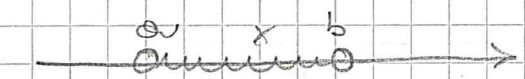
2 soluzioni coincidenti

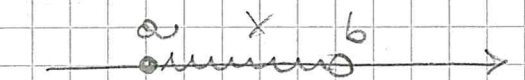
# INTERVALLI

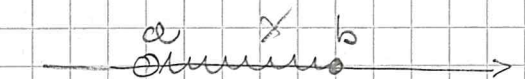
FASCE DI VALORI

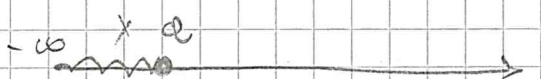
CONDIZIONI

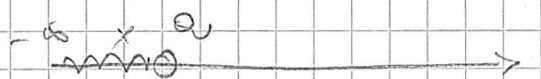

 $[a; b]$ 
 $a \leq x \leq b$

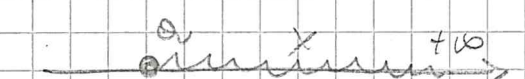

 $]a; b] = (a; b]$ 
 $a < x \leq b$

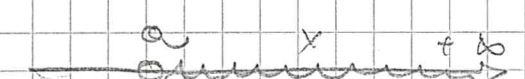

 $[a; b[ = [a; b)$ 
 $a \leq x < b$



 $]a; b[ = (a; b)$ 
 $a < x < b$

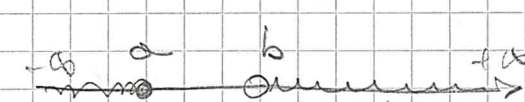

 $]-\infty; a] = (-\infty; a]$ 
 $x \leq a$

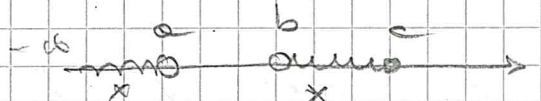

 $] -\infty; a[ = (-\infty; a)$ 
 $x < a$


 $[a; +\infty[ = [a; +\infty)$ 
 $x \geq a$

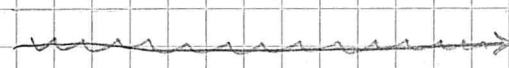

 $]a; +\infty[ = (a; +\infty)$ 
 $x > a$

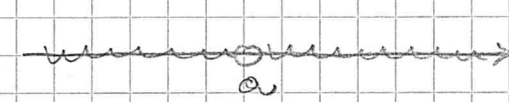

 $[a; b] \cup [c; +\infty[$ 
 $a \leq x \leq b \vee x \geq c$


 $]-\infty; a[ \cup ]b; +\infty[$ 
 $x < a \vee x > b$


 $]-\infty; a[ \cup ]b; c[$ 
 $x < a \vee b < x < c$

## Intervalli particolari:


 $\mathbb{R} = ]-\infty; +\infty[$


 $\mathbb{R} \setminus \{a\}$ 
 $x \neq a$


 $x = a$

$a \leq x \leq b \wedge x \neq c$

$a \leq x \leq b \vee x = c$

$\emptyset$   ~~$x \in \mathbb{R}$~~

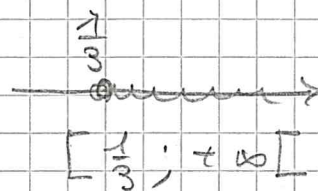


# DISEQUAZIONI DI 1°

$$3x - 1 \geq 0$$

$$\frac{3x}{3} \geq \frac{1}{3}$$

$$x \geq \frac{1}{3}$$



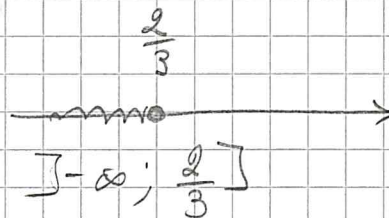
$$2 - 3x \geq 0$$

$$-3x \geq -2$$

cambio segno e  
cambio verso

$$\frac{3x}{3} \leq \frac{2}{3}$$

$$x \leq \frac{2}{3}$$

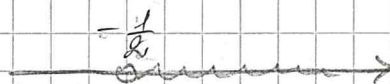


$$-\frac{1}{2} - x < 0$$

$$-x < +\frac{1}{2}$$

cambio segno e  
cambio verso

$$x > -\frac{1}{2}$$



$$-\frac{1}{2} - \frac{x}{3} \leq 0$$

$$\frac{-3 - 2x}{3} \leq \frac{0}{3}$$

$$-2x < +3$$

cambio segno e  
cambio verso

$$2x > -3$$

$$\frac{2x}{2} > \frac{-3}{2}$$


$$x > -\frac{3}{2}$$




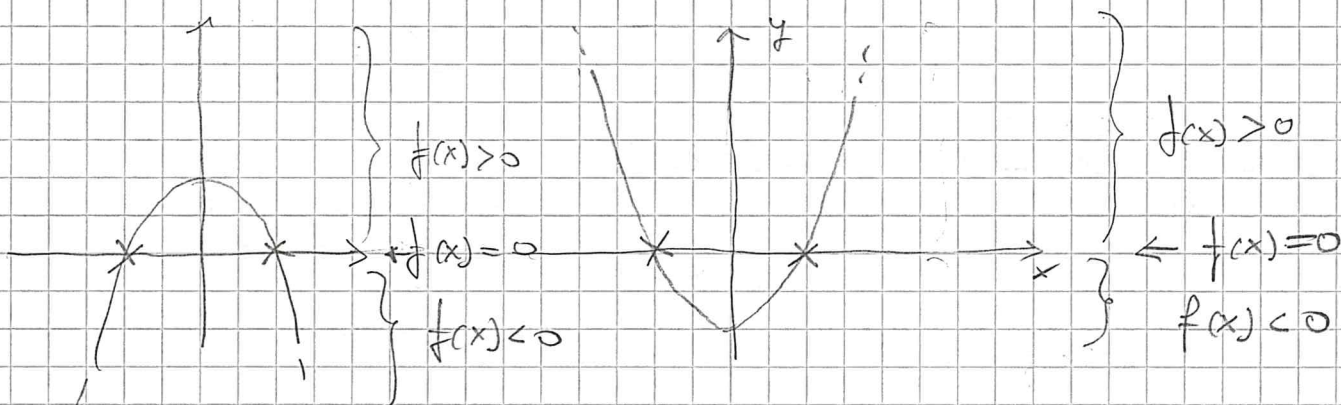
# DISEGNARE IONI DI 2°

## USANDO IL METODO GRAFICO DELLA PARABOLA

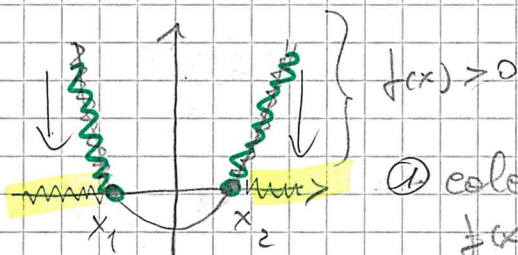
$$y = ax^2 + bx + c \quad \text{con } a \neq 0 \quad \text{parabola}$$

se  $a > 0 \Rightarrow$   parabola e  
rivolta  
verso l'alto

se  $a < 0 \Rightarrow$   parabola  
rivolta  
verso il basso



calcolo  $f(x) \geq 0$



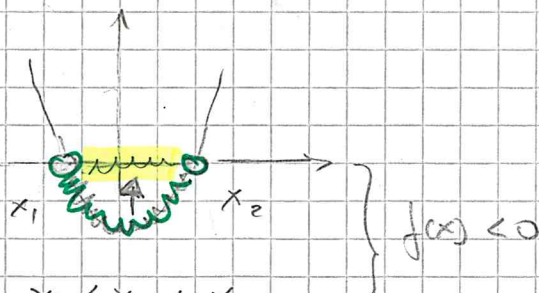
$$x \leq x_1 \vee x \geq x_2$$
$$\left] -\infty; x_1 \right] \cup \left[ x_2; +\infty \right[$$

① calcolo quando  $f(x) = 0$  e trovi  $x_1$  e  $x_2$

② evidenzia i tratti di parabole corrispondenti a  $f(x) \geq 0$

③ cerco le  $x$  in relazione con i tratti di parabole evidenziati

calcolo  $f(x) < 0$

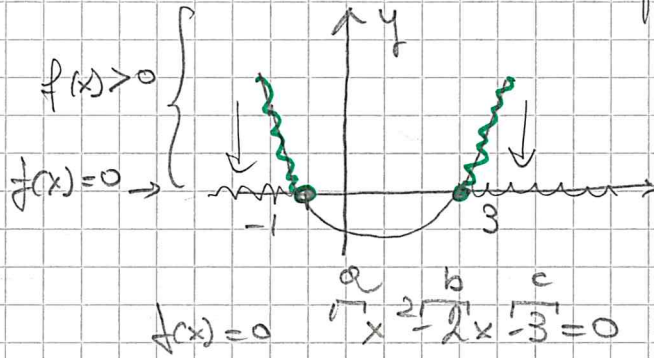


$$x_1 < x < x_2$$
$$\left] x_1; x_2 \right[$$



ES  $x^2 - 2x - 3 \geq 0$

$f(x) = x^2 - 2x - 3$  parabole 'U'   
 p. de  $a=1 > 0$



Sol:  $x \leq -1 \vee x \geq 3$   
 $] -\infty; -1] \cup [3; +\infty[$

$f(x) = 0$   
 $x^2 - 2x - 3 = 0$   
 $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$

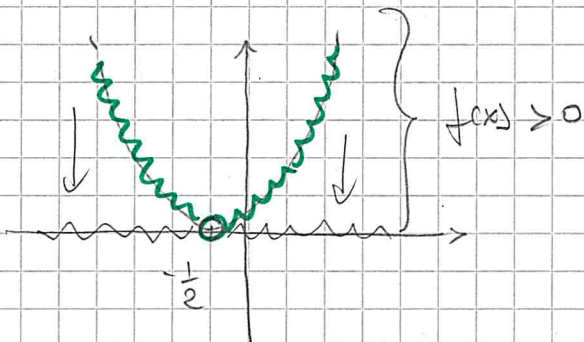
$= \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm \sqrt{16}}{2}$

$= \frac{2 \pm 4}{2} < \frac{-2}{2} = -1$

$\frac{6}{2} = 3$

ES  $4x^2 + 4x + 1 > 0$

$f(x) = 4x^2 + 4x + 1$  'U'  $a=4 > 0$



$f(x) = 0$   
 $4x^2 + 4x + 1 = 0$   
 $x_{1,2} = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(1)}}{2(4)}$

$= \frac{-4 \pm \sqrt{16 - 16}}{8} = \frac{-4 \pm \sqrt{0}}{8}$

Sol:  $x \neq -\frac{1}{2}$

$= \frac{-4}{8} = -\frac{1}{2}$

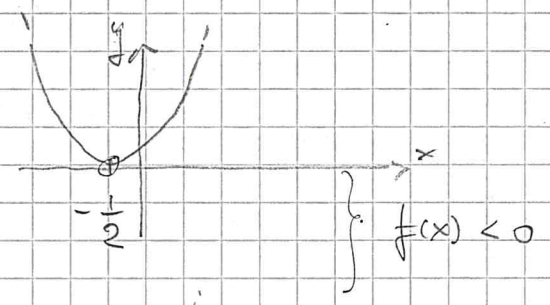
ES  $4x^2 + 4x + 1 \geq 0$

Sol:  $\mathbb{R}$



ES  $4x^2 + 4x + 1 < 0$

Sol: impossibile



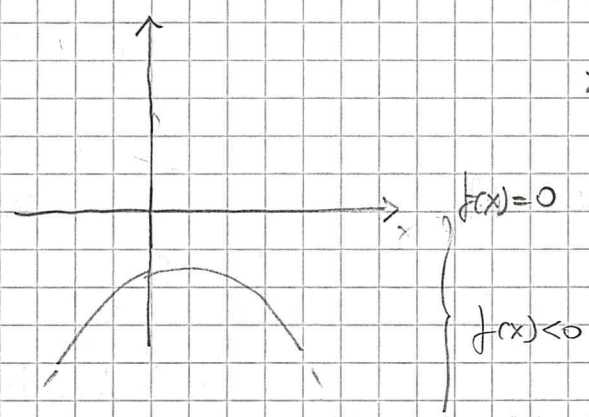
ES  $4x^2 + 4x + 1 \leq 0$

Sol  $x = -\frac{1}{2}$



ES  $-x^2 + x - 1 > 0$   
 $\geq 0$

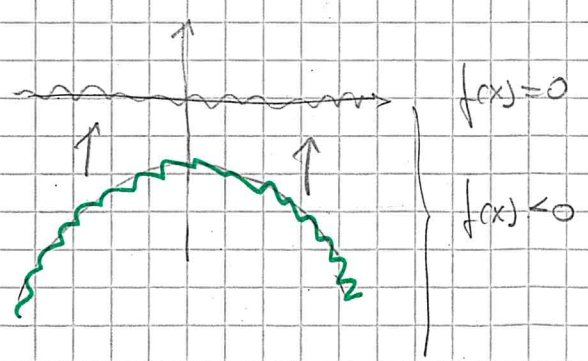
parabole  $\cap$  perché  $a = -1 < 0$   
 $f(x) = 0$   
 $\frac{a}{-1}x^2 + \frac{b}{+1}x + \frac{c}{-1} = 0$   
 $x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)(-1)}}{2(-1)}$   
 $= \frac{-1 \pm \sqrt{1-4}}{-2} = \frac{-1 \pm \sqrt{-3}}{-2}$   
 non ci sono le soluzioni  $x_1, x_2$



Sol: impossibile

ES  $-x^2 + x - 1 < 0$   
 $\leq 0$

come sopra



Sol:  $\mathbb{R}$



ES  $-x^2 + x \leq 0$

$f(x) = 0$   
 $a = -1 < 0$

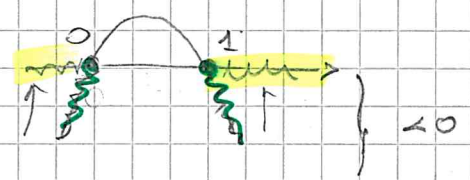
$-x^2 + x = 0$

$x(-x+1) = 0$

$x = 0 \vee -x+1 = 0$

$-x = -1$   
 $x = 1$

MS.  
con  
eq. di 2°  
INCOMPLETE



$x \leq 0 \vee x \geq 1$

ES  $-x^2 + x > 0$

come sopra  $x_1 = 0 \vee x_2 = 1$



$0 < x < 1$

ES  $x^2 - 4 > 0$

$f(x) = 0$   
 $a = 1 > 0$

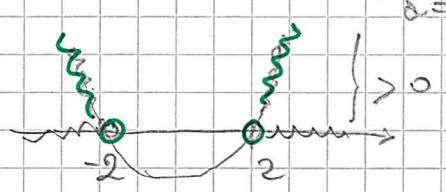
$f(x) = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

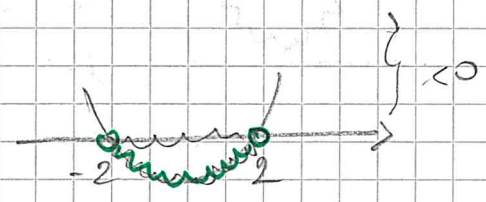
$x_1 = 2$   
 $x_2 = -2$



$x < -2 \vee x > 2$

ES  $x^2 - 4 < 0$

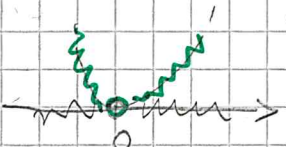
come sopra  $x_1 = 2 ; x_2 = -2$



$-2 < x < 2$



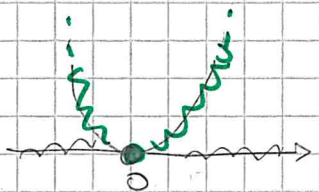
ES  $x^2 > 0$   $f(x) = 0$   $x^2 = 0$   $x_{1,2} = 0$



$y > 0$

Sol:  $x \neq 0$

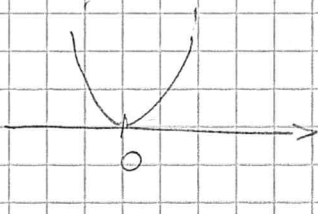
ES  $x^2 \geq 0$



$y > 0$   
 $y = 0$

Sol:  $\mathbb{R}$

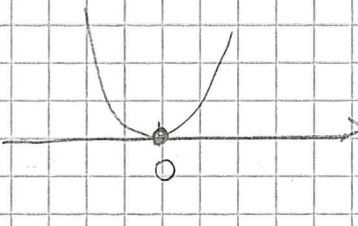
ES  $x^2 < 0$



$y < 0$

Sol: impossibile

ES  $x^2 \leq 0$

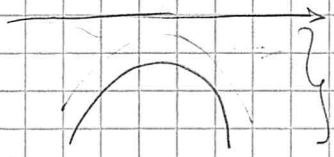


$y = 0$   
 $y < 0$

Sol:  $x = 0$

ES  $-x^2 - 1 \geq 0$   
 $> 0$

$f(x) = 0$   
 $-x^2 - 1 = 0$   
 $-x^2 = 1$   
 $x^2 = -1$   
 $x_{1,2} = \pm \sqrt{-1}$   
eq. imposs.

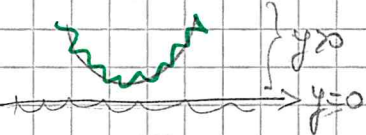


$y = 0$   
 $y < 0$

Sol: impossibile.

ES  $x^2 + 1 > 0$   
 $\geq 0$

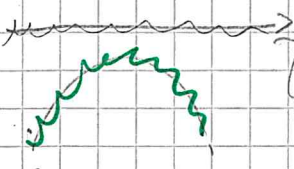
$f(x) = 0$   
 $x^2 + 1 = 0$   
 $x^2 = -1$   
 $x = \pm \sqrt{-1}$   
eq. impo



$y > 0$   
 $y = 0$

Sol:  $\mathbb{R}$

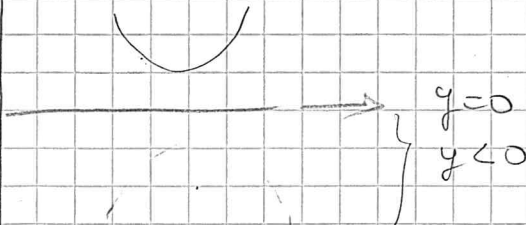
ES  $-x^2 - 1 < 0$   
 $\leq 0$



$y = 0$   
 $y < 0$

Sol:  $\mathbb{R}$

ES  $x^2 + 1 < 0$   
 $\leq 0$



$y = 0$   
 $y < 0$

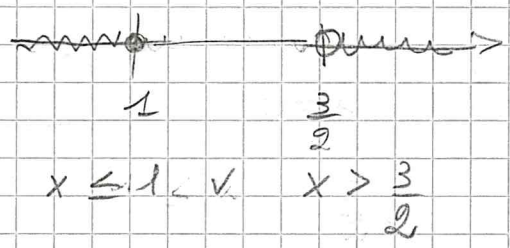
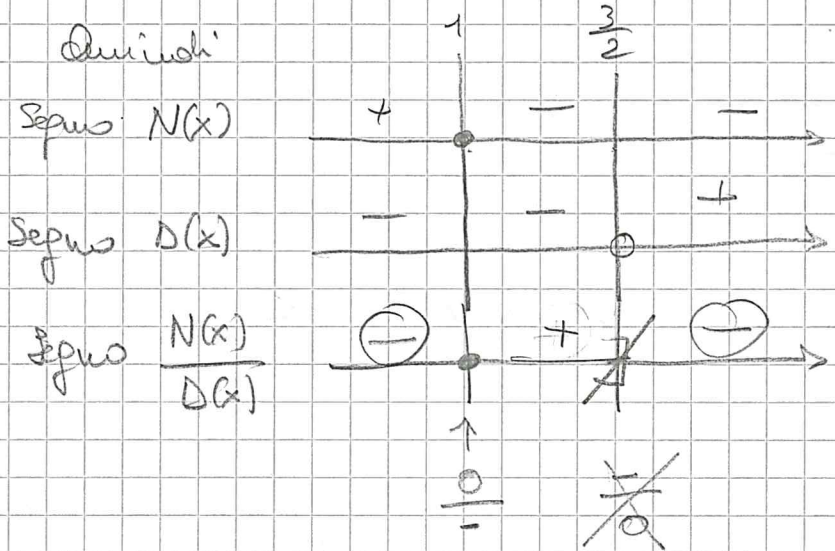
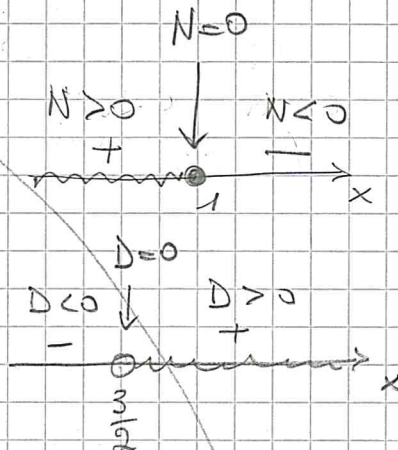
Sol: impossibile

# DI SEGNAZIONI FRATTE

ES  $\frac{1-x}{2x-3} \leq 0$

Segno  $N(x) \Rightarrow N(x) \geq 0 \quad 1-x \geq 0 \quad -x \geq -1 \quad x \leq 1$

Segno  $D(x) \Rightarrow D(x) > 0 \quad 2x-3 > 0 \quad \cancel{2x} > 3 \quad \cancel{2} > \frac{3}{2} \quad x > \frac{3}{2}$





BS

$$\frac{2-x}{x-x^2} < 0$$

$$N(x) > 0 \quad 2-x > 0 \Rightarrow x > -2 \quad x < 2$$

$$D(x) > 0 \quad x-x^2 > 0$$

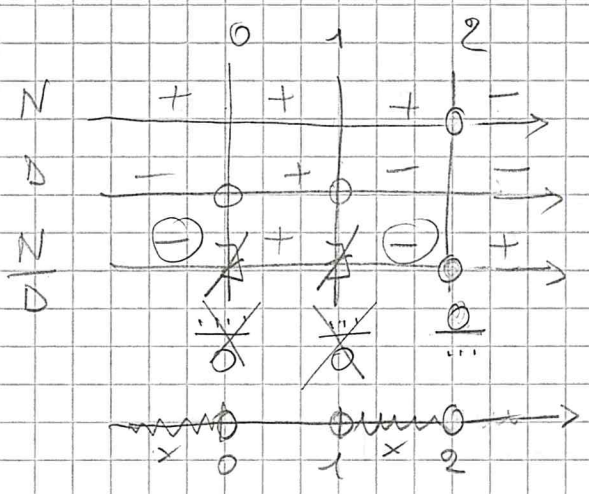
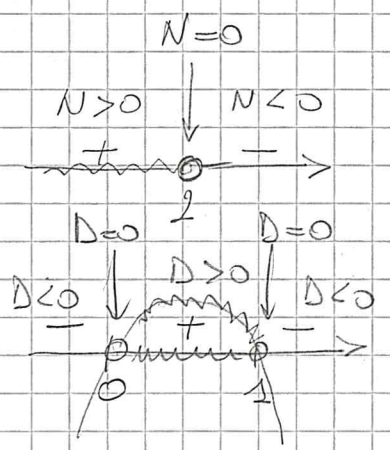
$$x-x^2=0$$

$$x(1-x)=0$$

$$x=0 \vee 1-x=0$$

$$-x=-1$$

$$x=1$$



$$x < 0 \vee 1 < x < 2$$

BS

$$\frac{x^2}{x^2-1} > 0$$

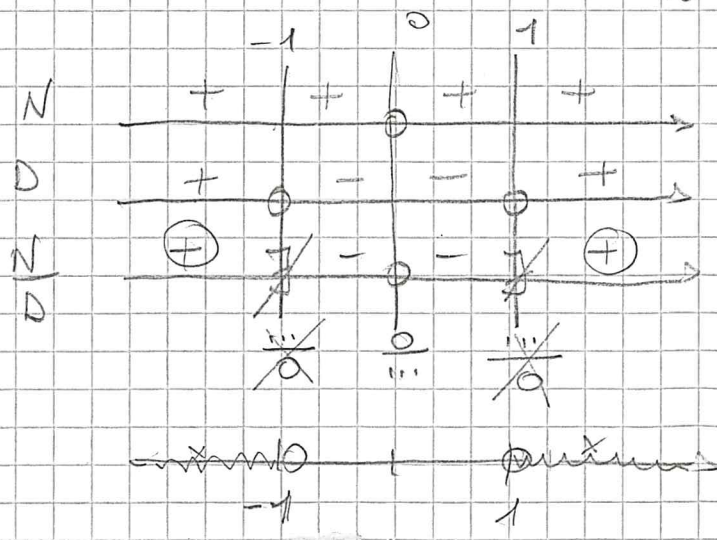
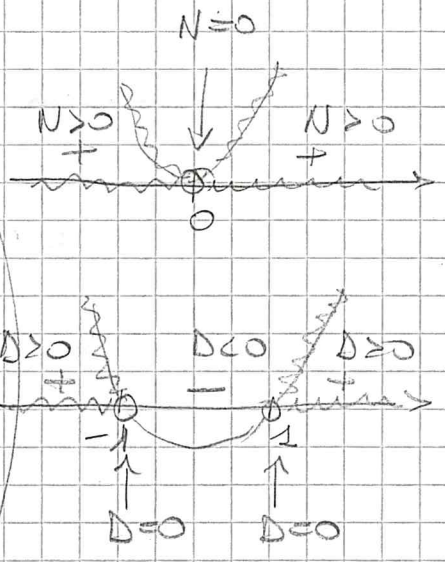
$$N(x) > 0 \quad x^2 > 0 \quad x^2=0 \quad x=0$$

$$D(x) > 0 \quad x^2-1 > 0$$

$$x^2-1=0$$

$$x^2=1$$

$$x_{1,2} = \pm 1$$



$$x < -1 \vee x > 1$$



SISTEMI DI INEQUAZIONI

$$\begin{cases} \textcircled{1} & x^2 - 4x + 3 \geq 0 \\ \textcircled{2} & x^2 - 2x < 0 \end{cases}$$

$$\textcircled{1} \quad \underbrace{x^2 - 4x + 3}_{y} \geq 0$$

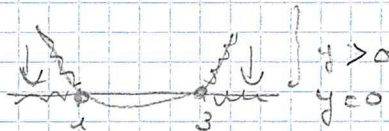
PARABOLA  $\cup$

$$x^2 - 4x + 3 = 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{16 - 4(1)(3)}}{2} = \frac{4 \pm 2}{2}$$

$$\frac{4-2}{2} = \frac{2}{2} = 1$$

$$\frac{4+2}{2} = \frac{6}{2} = 3$$



$$x \leq 1 \vee x \geq 3$$

$$\textcircled{2} \quad \underbrace{x^2 - 2x}_{y} < 0$$

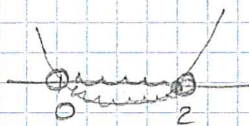
PARABOLA  $\cup$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0$$

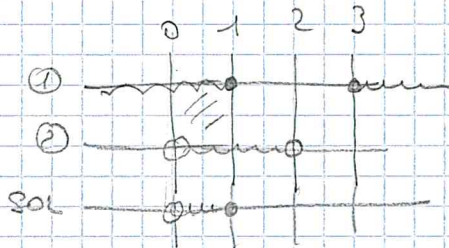
$$x-2 = 0 \quad x = 2$$



$$0 < x < 2$$

$$y < 0$$

$$\begin{cases} \textcircled{1} & x \leq 1 \vee x \geq 3 \\ \textcircled{2} & 0 < x < 2 \end{cases}$$



SOL:  $0 < x \leq 1$

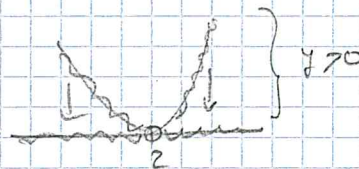
$$\begin{cases} \textcircled{1} & x^2 - 4x + 4 > 0 \\ \textcircled{2} & x^2 + 3x + 5 > 0 \end{cases}$$

$$\textcircled{1} \quad \underbrace{x^2 - 4x + 4}_{y} > 0$$

PARABOLA  $\cup$

$$x^2 - 4x + 4 = 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{16 - 4(1)(4)}}{2} = \frac{4}{2} = 2$$



$$x \neq 2$$

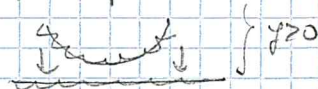
$$\textcircled{2} \quad \underbrace{x^2 + 3x + 5}_{y} > 0$$

PARABOLA  $\cup$

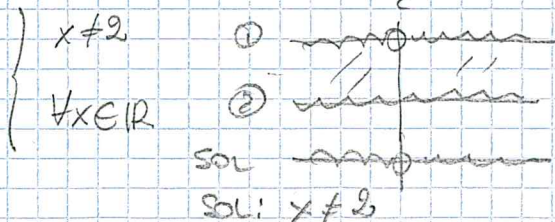
$$x^2 + 3x + 5 = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9 - 4(1)(5)}}{2} = \frac{-3 \pm \sqrt{-11}}{2}$$

epi. impossibile



$$\forall x \in \mathbb{R}$$



SOL

SOL:  $x \neq 2$



## Equazioni di 2° grado

1)  $10t^2 + 3t - 1 = 0$

$$\left[-\frac{1}{2}; \frac{1}{5}\right]$$

2)  $z^2 + 3z + 3 = 0$

$$[\nexists z \in \mathbb{R}] \rightarrow \text{impossibile}$$

## Diseguaglianze di 2°

1)  $5x \geq 2 + 2x^2$

$$\left[\frac{1}{2} \leq x \leq 2\right]$$

2)  $y^2 - 6y < -9$

$$[\nexists y \in \mathbb{R}]$$

3)  $t^2 > t - 5$

$$[\forall t \in \mathbb{R}]$$

4)  $\frac{x^2 - 1}{2x - x^2} \geq 0$

$$[-1 \leq x < 0 \vee 1 < x \leq 2]$$

5)  $\frac{x^2}{x^2 - 4} \geq 0$

$$[x < -2 \vee x = 0 \vee x > 2]$$

6)  $\frac{x^2 - 3x}{x^2 + 2} > 0$

$$[x < 0 \vee x > 3]$$

7)  $\frac{5x^2 - 4x}{x^2 - 6x + 9} \leq 0$

$$\left[0 < x \leq \frac{4}{5}\right]$$

# Sistemi

$$1) \begin{cases} x^2 - 4x + 3 \geq 0 \\ x^2 - 2x < 0 \end{cases} \quad [0 < x < 1]$$

$$2) \begin{cases} x^2 - 4x + 4 > 0 \\ x^2 + 3x + 5 > 0 \end{cases} \quad [x \neq 2]$$

$$3) \begin{cases} x^2 - 5x + 6 < 0 \\ x^2 + 3x - 4 > 0 \end{cases} \quad [2 < x < 3]$$

$$4) \begin{cases} 6x^2 + 7x + 2 < 0 \\ 4x^2 - 3x + 15 > 0 \end{cases} \quad \left[-\frac{2}{3} < x < -\frac{1}{2}\right]$$

$$\frac{-x^2}{1-2x+x^2} < 0$$

$$\frac{-x^2}{1-2x+x^2} \leq 0$$

$$\frac{-2}{x^2+1} \leq 0 \quad [\forall x \in \mathbb{R}]$$

$$\frac{-2}{x^2+1} \geq 0 \quad [\nexists x \in \mathbb{R}]$$