

## Esercizi sulle disequazioni esponenziali e logaritmiche

1.  $2^x > \frac{1}{16}$
2.  $\left(\frac{1}{3}\right)^x > 9$
3.  $4^{5x-1} < 2$
- 4.  $81^x \leq \frac{1}{3}$
5.  $3^{-x^2+4x} > 81$
6.  $5^{2(x-2)}(5^{2(x-1)})^{x+1} > 125^{x-1}$
- 7.  $5^{\frac{1}{x^2}} > 1$  [ $x \neq 0$ ]
8.  $3^{\frac{1}{x}} 3^{\frac{1}{x+1}} < 3$
9.  $2^{\frac{x}{x+1}} > 1$
10.  $\left(\left(\frac{1}{7}\right)^{x+1}\right)^x > \frac{1}{49}$
11.  $(3^{x-1})^{x+1} > 27$
- 12.  $\left(\frac{1}{2}\right)^{x^2+1} > \left(\frac{1}{2}\right)^2$
13.  $\left(\frac{\sqrt{2}}{2}\right)^{9x} > \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{x}}$
14.  $\log(x^2 + 1) > \log(2x + 4)$  [ $-\frac{3}{2} < x < -1$  e  $x > 3$ ]
15.  $2 \log_5 x \geq 3$  [ $x \geq \sqrt{125}$ ]
16.  $\log(2x - 3) + \log(x - 1) > 0$
17.  $\log x - \log 3 < \log(x + 2)$  [ $x > 0$ ]
18.  $\log_3(x + 1) + \log x < \log(5x - 3)$  [ $1 < x < 3$ ]
19.  $\log_{\frac{1}{2}}(x + 1) + \log_{\frac{1}{2}}(6x - 2) - \log_{\frac{1}{2}}(5x + 1) > \log_{\frac{1}{2}} 4$  [ $\frac{1}{2} < x < 3$ ]
20.  $\frac{1}{3} \log(x + 1) < 0$
21.  $\log_3 \log_3(4x + 6) < 0$
22.  $\text{Log}(2x^2 - x) < 0$
23.  $\text{Log}(x^2 - 13x - 14) > 2$
24.  $\log_{\frac{1}{5}} x \geq -1$

# Diseguazioni esponenziali

File  
diseg-log-pol

1)  $2^x > \frac{1}{16}$

$$2^x > \frac{1}{2^4}$$

$$2^x > 2^{-4}$$

base  $a=2 > 1$

$\Rightarrow$  mantenendo il verso

$$x > -4$$

3)  $4^{5x-1} < 2$

$$(2^2)^{5x-1} < 2$$

$$2^{2 \cdot (5x-1)} < 2^1$$

base  $a=2 > 1$

$\Rightarrow$  mantenendo il verso

$$2(5x-1) < 1$$

$$10x - 2 < 1$$

$$10x < 1 + 2$$

$$\frac{10x}{10} < \frac{3}{10}$$

$$x < \frac{3}{10}$$

5)  $3^{-x^2+4x} > 81$

$$3^{-x^2+4x} > 3^4$$

base  $a=3 > 1 \Rightarrow$

$\Rightarrow$  mantenendo il verso

$$-x^2 + 4x > 4$$

$$-x^2 + 4x - 4 > 0$$

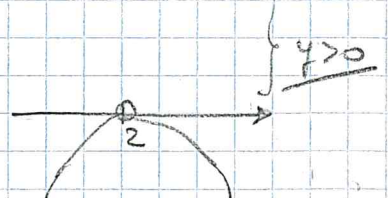
risolvo usando la

parabola che in questo caso è

$$-x^2 + 4x - 4 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(-1)(-4)}}{2(-1)} = \frac{-4 \pm \sqrt{16 - 16}}{-2} = \frac{-4}{-2} = 2$$



nessuna soluzione

2)  $(\frac{1}{3})^x > 9$

$$(3^{-1})^x > 3^2$$

$$3^{-x} > 3^2$$

base  $a=3 > 1$

$\Rightarrow$  mantenendo il verso

$$-x > 2$$

$$x < -2$$

4)  $81^x \leq \frac{1}{3}$

$$(3^4)^x \leq 3^{-1}$$

$$3^{4x} \leq 3^{-1}$$

base  $a=3 > 1$

$\Rightarrow$  mantenendo il verso

$$\frac{4x}{4} \leq \frac{-1}{4}$$

$$x \leq -\frac{1}{4}$$

$$6) 5^{2(x-2)} \cdot (5^{2(x-1)})^{x+1} > 125^{x-1}$$

$$5^{2(x-2)} \cdot 5^{2(x-1)(x+1)} > (5^3)^{x-1}$$

$$5^{2(x-2)} \cdot 5^{2(x^2-1)} > 5^{3(x-1)}$$

$$5^{2(x-2)+2(x^2-1)} > 5^{3(x-1)}$$

base  $a=5 > 1 \Rightarrow$  mantengo il verso

$$2(x-2)+2(x^2-1) > 3(x-1)$$

$$2x-4+2x^2-2 > 3x-3$$

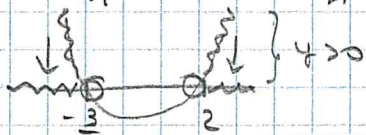
$$2x^2-x-6 > 0$$

risolvo usando la parabola che in questo caso è rivolta verso l'alto

$$2x^2-x-6=0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2-4(2)(-6)}}{2(2)} = \frac{1 \pm \sqrt{1+48}}{4} = \frac{1 \pm 7}{4}$$

Soluzioni:  $x < \frac{3}{2} \vee x > 2$



$$\frac{8}{4} = 2$$

$$\frac{-6}{2} = -3$$

7)  $5^{\frac{1}{x^2}} > 1$

NB serivo  $1=5^0$

base  $a=5 > 1 \Rightarrow$  mantengo il verso

$$5^{\frac{1}{x^2}} > 5^0$$

$$\frac{1}{x^2} > 0$$

$$x^2 > 0$$

risolvo usando la parabola

$$x^2 = 0 \Rightarrow x = 0$$

Soluzioni:  $x \neq 0$

8)  $3^{\frac{1}{x}} \cdot 3^{\frac{1}{x+1}} < 3$

$$3^{\frac{1}{x} + \frac{1}{x+1}} < 3^1$$

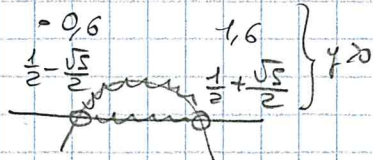
base  $a=3 > 1 \Rightarrow$  mantengo il verso

$$\frac{1}{x} + \frac{1}{x+1} < 1$$

$$\frac{1}{x} + \frac{1}{x+1} - 1 < 0$$

$$\frac{x+1+x-x(x+1)}{x(x+1)} < 0$$

$$\frac{x+1+x-x^2-x}{x(x+1)} < 0$$



$$\frac{-x^2+x+1}{x(x+1)} < 0$$

$N > 0 \Rightarrow -x^2+x+1 > 0$  uso la parabola

$D_1 > 0 \Rightarrow x > 0$

$D_2 > 0 \Rightarrow x+1 > 0 \Rightarrow x > -1$

$$-x^2+x+1=0 \quad x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1-4(-1)(+1)}}{2(-1)} = \frac{-1 \pm \sqrt{1+4}}{-2} = \frac{-1 \pm \sqrt{5}}{-2}$$

N	-	-	+	+	+
D <sub>1</sub>	-	-	-	+	+
D <sub>2</sub>	+	+	+	+	+
F	-	+	-	+	-

Soluzioni  $x < -1 \vee \frac{1}{2} - \frac{\sqrt{5}}{2} < x < 0 \vee x > \frac{1}{2} + \frac{\sqrt{5}}{2}$

9)  $2^{\frac{x}{x+1}} > 1$  NB  $1=2^0$

$2^{\frac{x}{x+1}} > 2^0$

base  $a=2 > 1 \Rightarrow$  mantenendo il verso

$\frac{x}{x+1} > 0$

$N > 0 \quad x > 0$

$D > 0 \quad x+1 > 0 \quad x > -1$

N  $\frac{0}{-} \quad \frac{-1}{+}$

D  $\frac{-}{-} \quad \frac{+}{+}$

F  $\frac{+}{+} \quad \frac{-}{+}$

Soluz.

$\frac{+}{-} \quad \frac{-}{+}$

Soluzioni:  $x < 0 \vee x > -1$

11)  $(3^{x-1})^{x+1} > 27$

$3^{(x-1)(x+1)} > 3^3$

base  $a=3 > 1 \Rightarrow$  mantenendo il verso

$(x-1)(x+1) > 3$

$x^2 - 1 > 3$

$x^2 - 5 > 0$

uso la parabola  $\cup \quad x^2 - 5 = 0$



$x^2 = 5$   
 $x = \pm\sqrt{5}$

Soluzioni:  $x < -\sqrt{5} \vee x > \sqrt{5}$

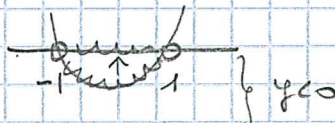
12)  $(\frac{1}{2})^{x^2+1} > (\frac{1}{2})^2$

base  $a = \frac{1}{2} < 1 \Rightarrow$  cambio il verso

$x^2 + 1 < 2$

$x^2 - 1 < 0$

uso la parabola  $\cup \quad x^2 - 1 = 0$   
 $x^2 = 1 \quad x = \pm 1$



Soluzioni:  $-1 < x < 1$

13)  $(\frac{\sqrt{2}}{2})^{9x} > (\frac{\sqrt{2}}{2})^{\frac{1}{x}}$

base  $a = \frac{\sqrt{2}}{2} < 1 \Rightarrow$  cambio il verso

$9x > \frac{1}{x}$

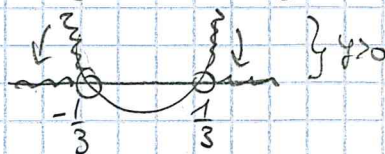
$9x - \frac{1}{x} > 0$

$\frac{9x^2 - 1}{x} > 0$

$N > 0 \quad 9x^2 - 1 > 0$  uso la parabola  $\cup$

$D > 0 \quad x > 0$

$9x^2 - 1 = 0$   
 $\frac{9x^2}{9} = \frac{1}{9} \Rightarrow x = \pm\sqrt{\frac{1}{9}} \Rightarrow x = \pm\frac{1}{3}$



N  $\frac{-1/3}{+} \quad \frac{0}{-} \quad \frac{1/3}{+}$

D  $\frac{-}{-} \quad \frac{+}{+}$

Soluzioni:  $-\frac{1}{3} < x < 0 \vee x > \frac{1}{3}$