Si calcolino i seguenti limiti:

1. $\lim_{x \to +\infty} x^{50} + 8x - 2^x$

2. $\lim_{x \to +\infty} \dfrac{x^{30} + 12x^{29} + 7}{x^{41} - 258x^{40}}$

3. $\lim_{x \to -\infty} -7x^3 + 5x - 9$

4 $\lim_{x \to +\infty} \dfrac{x^5 + 9x - 7}{x^2 - 3x^3}$

$\lim_{x \to 0^-} \dfrac{-x^2 + 3x - 1}{x^2 - 7x}$

6. $\lim_{x \to +\infty} \dfrac{-2x^3 + 5x - 8}{x^3 + x^2}$

7 $\lim_{x \to +\infty} x^2 - \log x^4$

8 $\lim_{x \to 0^+} 3x - \log x$

9. $\lim_{x \to -\infty} \dfrac{x^5 - 7x + 10}{x^3 + x^2 - 5}$

10. $$\lim_{x \to -\infty} \frac{x^2 - x + 1}{x - 4x^2 + 7}$$

11. $$\lim_{x \to 0^+} \frac{x^2 - x + 1}{x + 3}$$

12. $$\lim_{x \to 2^+} \frac{x^4 - 3x + 1}{x^2 - 3x + 2}$$

13. $$\lim_{x \to +\infty} \log \frac{x^3 - 1}{x^3 + 1}$$

14. $$\lim_{x \to 2^-} \log \frac{1}{x - 2}$$

15. $$\lim_{x \to +\infty} e^{\frac{x^2 - x - 1}{x^3 + 5x}}$$

16. $$\lim_{x \to 1^-} e^{\frac{x^2 + 2x + 1}{x^2 + 3x - 4}}$$

17. $$\lim_{x \to 3^+} \frac{x - 10}{x^2 + x - 6}$$

Esercizi - limiti.pdf

• 5) $\lim_{x \to 0^-} \frac{-x^2+3x-1}{x^2-7x} = \frac{-1}{0^+} = -\infty$

$x(x-7) = 0^- \cdot (-7) = 0^+$

oppure

$0^- = -0{,}1 \Rightarrow x^2 - 7x = (-0{,}1)^2 - 7(-0{,}1) = 0^+$

• 11) $\lim_{x \to 0^+} \frac{x^2-x+1}{x+3} = \frac{1}{3}$

• 12) $\lim_{x \to 2^+} \frac{x^4-3x+1}{x^2-3x+2} = \frac{16-6+1}{4-6+2} = \frac{11}{0^+} = +\infty$

$(x-2)(x-1) = 0^+ \cdot (1) = 0^+$

oppure

$2^+ = 2{,}1 \Rightarrow x^2-3x+2 = (2{,}1)^2 - 3(2{,}1) + 2 = 0^+$

17) 8) $\lim_{x \to 3^+} \frac{x-10}{x^2-x-6} = \frac{3-10}{9-3-6} = \frac{-7}{0^+} = -\infty$

$(x-3)(x+2) = 0^+ \cdot 5 = 0^+$

oppure

$3^+ = 3{,}1 \Rightarrow x^2-x-6 = (3{,}1)^2 - (3{,}1) - 6 = 0^+$

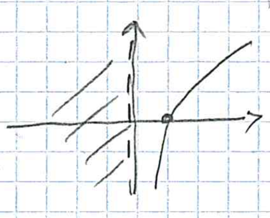
Ricordando che

la funzione esponenziale con base > 1

$\lim_{x \to +\infty} a^x = +\infty$

$\lim_{x \to -\infty} a^x = 0$

la funzione logaritmica con base > 1



$\lim_{x \to +\infty} \log_a x = +\infty$

$\lim_{x \to 0^+} \log_a x = -\infty$

• 8) $\lim_{x \to 0^+} 3x - \log x = 0 - \log(0^+) = 0 - (-\infty) = +\infty$

• 14) $\lim_{x \to 2^+} \log \frac{1}{x-2} = \log \frac{1}{0^+} = \log(+\infty) = +\infty$

• 16) $\lim_{x \to 1^-} e^{\frac{x^2+2x+1}{x^2+3x-4}} = e^{\frac{1+2+1}{1+3-4}} = e^{\frac{4}{0^-}} = e^{-\infty} = 0$

$(x+4)(x-1) = 5 \cdot (0^-) = 0^-$

oppure

$1^- = 0{,}9 \Rightarrow x^2+3x-4 = (0{,}9)^2 + 3(0{,}9) - 4 = 0^-$

3) $\lim\limits_{x\to-\infty} -7x^3+5x-9 = \lim\limits_{x\to-\infty} -7x^3 = -7(-\infty)^3 = -7(-\infty) = +\infty$

2) $\lim\limits_{x\to+\infty} \dfrac{x^{30}+12x^{29}+7}{x^{41}-258x^{40}} = \lim\limits_{x\to+\infty} \dfrac{\cancel{x^{30}}}{\cancel{x^{41}}\,{}^{11}} = \lim\limits_{x\to+\infty} \dfrac{1}{x^{11}} =$

$= \dfrac{1}{(+\infty)^{11}} = \dfrac{1}{+\infty} = 0$

4) $\lim\limits_{x\to+\infty} \dfrac{x^5+9x-7}{x^2-3x^3} = \lim\limits_{x\to+\infty} \dfrac{\cancel{x^5}^{\,3}}{\cancel{x^2}} = \lim\limits_{x\to+\infty} x^3 = (+\infty)^3 = +\infty$

6) $\lim\limits_{x\to+\infty} \dfrac{-2x^3+5x-8}{x^3+x^2} = \lim\limits_{x\to+\infty} \dfrac{-2\cancel{x^3}}{\cancel{x^3}} = -2$

9) $\lim\limits_{x\to-\infty} \dfrac{x^5-7x+10}{x^3+x^2-5} = \lim\limits_{x\to-\infty} \dfrac{\cancel{x^5}^{\,2}}{\cancel{x^3}} = \lim\limits_{x\to-\infty} x^2 = (-\infty)^2 = +\infty$

10) $\lim\limits_{x\to-\infty} \dfrac{x^2-x+1}{x-4x^2+7} = \lim\limits_{x\to-\infty} \dfrac{\cancel{x^2}}{-4\cancel{x^2}} = -\dfrac{1}{4}$

13) $\lim\limits_{x\to+\infty} \log\dfrac{x^3-1}{x^3+1} = \lim\limits_{x\to+\infty} \log\dfrac{\cancel{x^3}}{\cancel{x^3}} = \log 1 = 0$

15) $\lim\limits_{x\to+\infty} e^{\frac{x^2-x-1}{x^3+5x}} = \lim\limits_{x\to+\infty} e^{\frac{x^2}{x^3}} = \lim\limits_{x\to+\infty} e^{\frac{1}{x}} = e^{\frac{1}{+\infty}} = e^{0^+} = 1$

Gerarchia degli infiniti per $x\to+\infty$

l'infinito del logaritmo (con base >1) $<$ l'infinito della potenza (con esponente positivo) $<$ l'infinito dell'esponenziale (con base >1)

1) $\lim\limits_{x\to+\infty} x^{50}+8e-2^x = \lim\limits_{x\to+\infty} -2^x = -2^{+\infty} = -(+\infty) = -\infty$

7) $\lim\limits_{x\to+\infty} x^2-\log x^4 = \lim\limits_{x\to+\infty} x^2 = (+\infty)^2 = +\infty$