Integrali indefiniti (esercizi misti)
libro di Marco Abate

ES n. 7.9 (a, b, c)

a) $\int (6x-3)\,dx = 6\int x\,dx - 3\int dx = \overset{3}{\cancel{6}}\frac{x^2}{\cancel{2}} - 3x + c = 3x^2 - 3x + c$

b) $\int (ax^3 - 2ax + 2)\,dx = a\int x^3\,dx - 2a\int x\,dx + 2\int 1\,dx =$

con $a \in \mathbb{R}$ costante $= a\frac{x^4}{4} - \cancel{2}a\frac{x^2}{\cancel{2}} + 2x + c = \frac{a}{4}x^4 - ax^2 + 2x + c$

c) $\int \operatorname{sen}(2t)\,dt = \frac{1}{2}\int 2\operatorname{sen}(2t)\,dt = \frac{1}{2}\left(-\cos(2t)\right) + c$

$= -\frac{1}{2}\cos(2t) + c$

Integrali definiti (libro Marco Abate)

ES n 7.11

a) $\int_{-1}^{0} (3x^2 + 2x + 1)\,dx = \left[\cancel{3}\cdot\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}} + x\right]_{-1}^{0} = \left[x^3 + x^2 + x\right]_{-1=a}^{0=b} = F(0) - F(-1)$

$= 0 - \left[(-1)^3 + (-1)^2 + (-1)\right] = -\left[-1 + \cancel{1} - \cancel{1}\right] = 1$

b) $\int_{0}^{1} (4y^3 + 1)\,dy = \left[\cancel{4}\frac{y^4}{\cancel{4}} + y\right]_0^1 = \left[y^4 + y\right]_{0=a}^{1=b} = F(1) - F(0) =$

$= \left[(1)^4 + (1)\right] - (0) = 2$

c) $\int_0^1 (3t^2 - 1)\,dt = \left[\cancel{3}\cdot\frac{t^3}{\cancel{3}} - t\right]_{0=a}^{1=b} = F(1) - F(0) = \left[\cancel{1^3} - \cancel{1}\right] - 0 = 0$

d) $\int_{-2}^{2} (2s^3 + 3s - 4)\,ds = \left[\cancel{2}\frac{s^4}{\cancel{4}_2} + 3\frac{s^2}{2} - 4s\right]_{-2}^{2} = F(2) - F(-2) =$

$= \left[\frac{(2)^{\cancel{4}\,3}}{\cancel{2}} + \frac{3}{\cancel{2}}\cdot(2)^{\cancel{2}} - 4(2)\right] - \left[\frac{(-2)^4}{2} + \frac{3}{2}(-2)^2 - 4(-2)\right] =$

$= \left[\cancel{8} + 6 - \cancel{8}\right] - \left[\frac{\overset{8}{\cancel{16}}}{\cancel{2}} + \frac{3}{\cancel{2}}\cdot\overset{2}{\cancel{4}} + 8\right] = \cancel{6} - 8 - \cancel{6} - 8 = -16$

e) $\int_1^3 (-3z^4 + z^3 - 3z + 1)\,dz = \left[-3\frac{z^5}{5} + \frac{z^4}{4} - 3\frac{z^2}{2} + z\right]_1^3 = F(3) - F(1) =$

$= \left[-3\cdot\frac{(3)^5}{5} + \frac{(3)^4}{4} - 3\cdot\frac{(3)^2}{2} + 3\right] - \left[-3\frac{(1)^5}{5} + \frac{(1)^4}{4} - 3\frac{(1)^2}{2} + 1\right] =$

$= \left[-\frac{729}{5} + \frac{81}{4} - \frac{27}{2} + 3\right] - \left[-\frac{3}{5} + \frac{1}{4} - \frac{3}{2} + 1\right]$

$= \frac{-2916 + 405 - 270 + 60}{20} - \frac{-12 + 5 - 30 + 20}{20} = \frac{-2721 + 17}{20} = \frac{-\overset{676}{\cancel{2704}}}{\underset{5}{\cancel{20}}}$

$= -\frac{676}{5}$

ES n 7.12

a) $\int_0^1 (e^x + x^2)\,dx = \left[e^x + \frac{x^3}{3}\right]_0^1 = F(1) - F(0) = \left[e^1 + \frac{1}{3}\right] - \left[\underset{1}{e^0} + 0\right] = e + \frac{1}{3} - 1$

$= e - \frac{2}{3}$

b) $\int_1^2 \frac{1}{r^2}\,dr = \int_1^2 r^{-2}\,dr = \left[\frac{r^{-1}}{-1}\right]_1^2 = \left[-\frac{1}{r}\right]_1^2 = F(2) - F(1) =$

$= -\frac{1}{2} - \left[-\frac{1}{1}\right] = -\frac{1}{2} + 1 = +\frac{1}{2}$

c) $\int_0^2 (t + e^t)\,dt = \left[\frac{t^2}{2} + e^t\right]_0^2 = F(2) - F(0) = \left[\frac{(2)^{\cancel{2}}}{\cancel{2}} + e^2\right] - \left[0 + \underset{1}{e^0}\right] =$

$= 2 + e^2 - 1 = 1 + e^2$

d) $\int_{-1}^1 \frac{1}{2}(e^s + e^{-s})\,ds = \frac{1}{2}\int_{-1}^1 (e^s + e^{-s})\,ds =$

$= \frac{1}{2}\left(\int_{-1}^1 e^s\,ds + (-)\int_{-1}^1 \underbrace{-e^{-s}}_{f'}\,ds\right) = \frac{1}{2}\left[e^s - e^{-s}\right]_{-1}^1 = F(1) - F(-1)$

$= \frac{1}{2}\left(\left[e^1 - e^{-1}\right] - \left[e^{-1} - e^{-(-1)}\right]\right) = \frac{1}{2}\left(e - e^{-1} - e^{-1} + e\right)$

$= \frac{1}{2}\left(2e - 2e^{-1}\right) = e - e^{-1}$

f) $\int_{-3}^{-1} \left(\frac{2}{m^3} - \frac{2}{m}\right)dm = \int_{-3}^{-1}\left(2\cdot m^{-3} - \frac{2}{m}\right)dm = \left[\frac{\cancel{2}\cdot m^{-2}}{-\cancel{2}} - 2\ln|m|\right]_{-3}^{-1}$

$= \left[-\frac{1}{m^2} - 2\ln|m|\right]_{-3}^{-1} = F(-1) - F(-3) = \left[\frac{-1}{(-1)^2} - 2\ln|-1|\right] - \left[\frac{-1}{(-3)^2} - 2\ln|-3|\right]$

$= \left[-1 - \underbrace{\cancel{2\ln 1}}_{0}\right] - \left[-\frac{1}{9} - 2\ln 3\right] = -1 + \frac{1}{9} + 2\ln 3 = \frac{-8}{9} + 2\ln 3$

ES n 7.13 (a, b, c)

a) $\int_0^{\pi/2} \operatorname{sen} y\,dy = \left[-\cos y\right]_0^{\frac{\pi}{2}} = F\left(\frac{\pi}{2}\right) - F(0) = \left[-\cos\frac{\pi}{2}\right] - \left[-\cos 0\right] = (-0) - (-1) =$
$= 1$

b) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x\,dx = \left[\operatorname{sen} x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = F\left(\frac{\pi}{4}\right) - F\left(-\frac{\pi}{4}\right) = \left[\operatorname{sen}\frac{\pi}{4}\right] - \left[\operatorname{sen}\left(-\frac{\pi}{4}\right)\right] = \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

c) $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{1}{\cos^2(2t)}\,dt = \frac{1}{2}\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} 2\cdot\frac{1}{\cos^2(2t)}\,dt = \frac{1}{2}\left[\operatorname{tg}(2t)\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}} = \frac{1}{2}\left(F\left(\frac{\pi}{6}\right) - F\left(\frac{\pi}{8}\right)\right)$

$= \frac{1}{2}\left(\operatorname{tg}\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}_3}\right) - \operatorname{tg}\left(\cancel{2}\cdot\frac{\pi}{\cancel{8}_4}\right)\right) = \frac{1}{2}\left(\operatorname{tg}\frac{\pi}{3} - \operatorname{tg}\frac{\pi}{4}\right) = \frac{1}{2}\left(\sqrt{3} - 1\right)$

INTEGRAZIONE PER PARTI (esercizi vari / libro Marco Abate)

n. 18 (a; b; c) integrali indefiniti per parti $\quad \int f'(x)\, g(x)\, dx = \underbrace{f(x)}_{\int f'} \cdot g(x) - \int f(x) \cdot g'(x)$

a) $\displaystyle\int \underbrace{(2t-1)}_{g} \underbrace{\cos t}_{f'}\, dt \overset{\text{XPARTI}}{=} \underbrace{\operatorname{sen} t}_{\substack{f \\ \int f'}} \cdot \underbrace{(2t-1)}_{g} - \int \underbrace{\operatorname{sen} t}_{f} \cdot \underbrace{2}_{g'}\, dt =$

$= \operatorname{sen} t\,(2t-1) - 2\int \operatorname{sen} t\, dt =$

$= \operatorname{sen} t\,(2t-1) - 2\,(-\cos t) + c =$

$= \operatorname{sen} t\,(2t-1) + 2\cos t + c$

b) $\displaystyle\int (2t\cos t - t^2 \operatorname{sen} t)\, dt = 2\int t\cos t\, dt - \int t^2 \operatorname{sen} t\, dt = \text{III}$

I) $\displaystyle\int \underbrace{t}_{g} \underbrace{\cos t}_{f'}\, dt \overset{\text{XPARTI}}{=} \underbrace{\operatorname{sen} t}_{\substack{f \\ \int f'}} \cdot \underbrace{t}_{g} - \int \underbrace{\operatorname{sen} t}_{f} \cdot \underbrace{1}_{g'}\, dt = t \operatorname{sen} t - \int \operatorname{sen} t\, dt$

$= t\operatorname{sen} t - (-\cos t) + c$

$= t\operatorname{sen} t + \cos t + c$

II) $\displaystyle\int \underbrace{t^2}_{g} \underbrace{\operatorname{sen} t}_{f'}\, dt \overset{\text{XPARTI}}{=} \underbrace{-\cos t}_{\substack{f \\ \int f'}} \cdot \underbrace{t^2}_{g} - \int \underbrace{-\cos t}_{f} \cdot \underbrace{2t}_{g'} =$

$= -t^2\cos t + 2\underbrace{\int t\cos t\, dt}_{\text{già calcolato in I}} =$

$= -t^2\cos t + 2\,(t\operatorname{sen} t + \cos t) + c =$

$= -t^2\cos t + 2t\operatorname{sen} t + 2\cos t + c =$

$= (-t^2+2)\cos t + 2t\operatorname{sen} t + c$

III $= 2\cdot(t\operatorname{sen} t + \cos t) - \left[(-t^2+2)\cos t + 2t\operatorname{sen} t\right] + c =$

$= \cancel{2t\operatorname{sen} t} + 2\cos t - (-t^2+2)\cos t - \cancel{2t\operatorname{sen} t} + c =$

$= (\cancel{2} + t^2 - \cancel{2})\cos t + c = t^2\cos t + c$

c) $\displaystyle\int \underbrace{(2s^2-s)}_{g} \underbrace{e^s}_{f'}\, ds \overset{\text{XPARTI}}{=} \underbrace{e^s}_{\int f' = f} \cdot \underbrace{(2s^2-s)}_{g} - \int \underbrace{e^s}_{f} \cdot \underbrace{(2s-1)}_{g'}\, ds =$

$= (2s^2-s)e^s - \displaystyle\int \underbrace{(2s-1)}_{g} \underbrace{e^s}_{f'}\, ds \overset{\text{XPARTI}}{=}$

$= (2s^2-s)e^s - \left[\underbrace{e^s}_{\int f' = f} \cdot \underbrace{(2s-1)}_{g} - \displaystyle\int \underbrace{e^s}_{f} \cdot \underbrace{2}_{g'}\, ds\right] =$

$= (2s^2-s)e^s - (2s-1)e^s + 2\displaystyle\int e^s\, ds =$

$= (2s^2-s)e^s - (2s-1)e^s + 2e^s + c = e^s(2s^2 - s - 2s + 1 + 2) + c$

• d) $\int \underbrace{x^2}_{f'} \underbrace{\ln x}_{g} \, dx = \underbrace{\frac{x^3}{3}}_{\int f' = f} \cdot \underbrace{\ln x}_{g} - \int \underbrace{\frac{\cancel{x^3}^{\,x^2}}{3}}_{f} \cdot \underbrace{\frac{1}{\cancel{x}}}_{g'} \, dx = \frac{x^3}{3} \ln x - \int \frac{x^2}{3} \, dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + c = \frac{x^3}{3} \ln x - \frac{1}{9} x^3 + c$

ESn 7.19

• a) $\int_0^{2\pi} x^2 \cos x \, dx = (*)$

$\int \underbrace{x^2}_{g} \underbrace{\cos x}_{f'} \, dx \overset{\times \text{PARTI}}{=} \underbrace{\operatorname{sen} x}_{\int f' = f} \cdot \underbrace{x^2}_{g} - \int \underbrace{\operatorname{sen} x}_{f} \cdot \underbrace{2x}_{g'} \, dx =$

$= x^2 \operatorname{sen} x - 2 \int \underbrace{x}_{g} \underbrace{\operatorname{sen} x}_{f'} \, dx \overset{\times \text{PARTI}}{=}$

$= x^2 \operatorname{sen} x - 2 \left[\underbrace{-\cos x}_{\int f' = f} \cdot \underbrace{x}_{g} - \int \underbrace{-\cos x}_{f} \cdot \underbrace{1}_{g'} \, dx \right] =$

$= x^2 \operatorname{sen} x + 2 x \cos x - 2 \int \cos x \, dx =$

$= x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + c =$

$= (x^2 - 2) \operatorname{sen} x + 2 x \cos x + c$

Quindi

$(*) = \left[(x^2 - 2) \operatorname{sen} x + 2x \cos x \right]_0^{2\pi} = F(2\pi) - F(0) =$

$= \left[((2\pi)^2 - 2) \underbrace{\operatorname{sen}(2\pi)}_{=0} + 2(2\pi) \underbrace{\cos(2\pi)}_{1} \right] - \left[\cancel{(0-2) \underbrace{\operatorname{sen} 0}_{0} + 2 \cdot 0 \cdot \cos 0} \right] = 4\pi$

• b) $\int_1^2 t \sqrt{1+t} \, dt = (*)$

NB $\int \sqrt{1+t} \, dt = \int 1 \cdot (1+t)^{\frac{1}{2}} \, dt$

$= \frac{(1+t)^{\left(\frac{1}{2}+1\right) = \frac{3}{2}}}{\frac{3}{2}} + c =$

$= \frac{2}{3} (1+t)^{\frac{3}{2}} + c$

$\int \underbrace{t}_{g} \underbrace{\sqrt{1+t}}_{f'} \, dt = \underbrace{\frac{2}{3} (1+t)^{\frac{3}{2}}}_{\int f' = f} \cdot \underbrace{t}_{g} - \int \underbrace{\frac{2}{3} (1+t)^{\frac{3}{2}}}_{f} \cdot \underbrace{1}_{g'} \, dt$

$= \frac{2}{3} t \sqrt{(1+t)^3} - \frac{2}{3} \int (1+t)^{\frac{3}{2}} \, dt =$

$= \frac{2}{3} t \sqrt{(1+t)^3} - \frac{2}{3} \frac{(1+t)^{\frac{3}{2}+1 = \frac{5}{2}}}{\frac{5}{2}} + c$

$= \frac{2}{3} t \sqrt{(1+t)^3} - \frac{2}{3} \cdot \frac{2}{5} \sqrt{(1+t)^5} + c = \frac{2}{3} t \sqrt{(1+t)^3} - \frac{4}{15} \sqrt{(1+t)^5} + c$

Quindi

$(*) = \left[\frac{2}{3} t \sqrt{(1+t)^3} - \frac{4}{15} \sqrt{(1+t)^5} \right]_1^2 = F(2) - F(1) =$

$= \left[\frac{2}{3} (2) \sqrt{(1+2)^3} - \frac{4}{15} \sqrt{(1+2)^5} \right] - \left[\frac{2}{3} (1) \sqrt{(1+1)^3} - \frac{4}{15} \sqrt{(1+1)^5} \right] =$

$= \frac{4}{3} \sqrt{3^3} - \frac{4}{15} \sqrt{3^5} - \frac{2}{3} \sqrt{2^3} + \frac{4}{15} \sqrt{2^5} = \frac{4}{3} \cdot 3\sqrt{3} - \frac{4}{15} \cdot 3^2 \sqrt{3} - \frac{2}{3} \cdot 2\sqrt{2} + \frac{4}{15} \cdot 2^2 \sqrt{2} =$

$= 3\sqrt{3} - \frac{12}{5} \sqrt{3} - \frac{4}{3} \sqrt{2} + \frac{16}{15} \sqrt{2} =$

$= \frac{3}{5} \sqrt{3} - \frac{4}{15} \sqrt{2}$