INTEGRAZIONE PER PARTI

$$\int f'(x)\, g(x)\, dx = \underbrace{f(x)}_{\int f'(x)\,dx} \cdot g(x) - \int f(x) \cdot g'(x)$$

NB
$$\int f'(x)\, dx = f(x) + c$$

TIPO 1

$$\int x^n \overset{f'}{e^x}\, dx$$

$$\int x^n \overset{f'}{\operatorname{sen} x}\, dx$$

$$\int x^n \overset{f'}{\cos x}\, dx$$

TIPO 2

$$\int \overset{f'}{x^n} \ln x\, dx \qquad \int \overset{f'}{1} \cdot \ln x\, dx$$

$$\int \overset{f'}{x^n} \operatorname{arcsen} x\, dx \qquad \int \overset{f'}{1} \cdot \operatorname{arcsen} x\, dx$$

$$\int \overset{f'}{x^n} \arccos x\, dx \qquad \int \overset{f'}{1} \cdot \arccos x\, dx$$

$$\int \overset{f'}{x^n} \operatorname{arctg} x\, dx \qquad \int \overset{f'}{1} \cdot \operatorname{arctg} x\, dx$$

INTEGRALI CICLICI (TIPO 3)

$$\int e^x \operatorname{sen} x\, dx$$

$$\int e^x \cos x\, dx$$

$$\int \operatorname{sen}^2 x\, dx$$

$$\int \cos^2 x\, dx$$

TIPO 1

$$\int \underbrace{x}_{g}\,\underbrace{e^x}_{f'}\,dx = \underbrace{e^x}_{f}\cdot\underbrace{x}_{g} - \int \underbrace{e^x}_{f}\cdot\underbrace{1}_{g'}\,dx = xe^x - e^x + C$$

$$\int \underbrace{x^2}_{g}\,\underbrace{e^x}_{f'}\,dx = e^x\cdot x^2 - \int e^x\cdot 2x\,dx =$$

$$= x^2e^x - 2\int \underbrace{e^x}_{f'}\cdot x\,dx =$$

$$= x^2e^x - 2\left[e^x\cdot x - \int e^x\cdot 1\,dx\right]$$

$$= x^2e^x - 2\left[xe^x - e^x\right]$$

$$= x^2e^x - 2xe^x + 2e^x$$

$$\int \operatorname{sen} x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \operatorname{sen} x + C$$

$$\int \underbrace{x}_{g}\,\underbrace{\operatorname{sen} x}_{f'}\,dx = (-\cos x)\cdot x - \int (-\cos x)\cdot 1\,dx =$$

$$= -x\cos x + \int \cos x\,dx =$$

$$= -x\cos x + \operatorname{sen} x + c$$

$$\int \underbrace{x}_{g}\,\underbrace{\cos x}_{f'}\,dx = \operatorname{sen} x\cdot x - \int \operatorname{sen} x\cdot 1\,dx =$$

$$= x\operatorname{sen} x - (-\cos x) + c =$$

$$= x\operatorname{sen} x + \cos x + c$$

$$\int \underset{g}{x^2}\,\underbrace{\text{sen}\,x}_{f'}\,dx = (-\cos x)\cdot x^2 - \int(-\cos x)\cdot 2x\,dx =$$

$$= -x^2\cos x + 2\underbrace{\int \overset{g\,\cdot\,f'}{x\cos x}\,dx}_{\text{PER PARTI}} =$$

$$= -x^2\cos x + 2\left[\text{sen}\,x\cdot x - \int \text{sen}\,x\cdot 1\,dx\right] =$$

$$= -x^2\cos x + 2x\,\text{sen}\,x - 2\int \text{sen}\,x\,dx =$$

$$= x^2\cos x + 2x\,\text{sen}\,x - 2(-\cos x) + c$$

$$= x^2\cos x + 2x\,\text{sen}\,x + 2\cos x + c$$

$$\int \underset{g\,\cdot}{x^2}\,\underbrace{\cos x}_{f'}\,dx = \text{sen}\,x\cdot x^2 - \int \text{sen}\,x\cdot 2x\,dx =$$

$$= x^2\,\text{sen}\,x - 2\underbrace{\int \overset{g\,\cdot\,f'}{x\,\text{sen}\,x}\,dx}_{\text{PER PARTI}} =$$

$$= x^2\,\text{sen}\,x - 2\left[(-\cos x)\cdot x - \int(-\cos x)\cdot 1\,dx\right] =$$

$$= x^2\,\text{sen}\,x + 2x\cos x - 2\int \cos x\,dx =$$

$$= x^2\,\text{sen}\,x + 2x\cos x - 2\,\text{sen}\,x + c$$

TIPO 2

$$\int \underset{f'}{x} \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x^{\cancel{2}}}{2}\frac{1}{\cancel{x}}\, dx =$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx =$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + c = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

$$\int \ln x \, dx = \int \underset{f'}{1}\cdot \ln x \, dx = x \ln x - \int \cancel{x}\cdot\frac{1}{\cancel{x}}\, dx =$$

$$= x\ln x - \int 1 \cdot dx =$$

$$= x \ln x - x + c$$

$$\int \operatorname{arctg} x \, dx = \int \underset{f'}{1}\cdot \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \overset{*}{\int x \cdot \frac{1}{1+x^2}\, dx}$$

$$= x \operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + c$$

TIPO $\int \frac{f'}{f}\, dx$

* [NB $\int x' \cdot \frac{1}{1+x^2}\, dx = \frac{1}{2}\int \underset{f'}{2x}\cdot\frac{1}{1+x^2}\, dx = \frac{1}{2}\ln(1+x^2) + c$]

$$\int \operatorname{arcsen} x \, dx = \int 1 \cdot \operatorname{arcsen} x \, dx = x \operatorname{arcsen} x - \overset{**}{\int x \cdot \frac{1}{\sqrt{1-x^2}}\, dx}$$

$$=$$

tipo $\int f^n f'$

** [$\int x \cdot \frac{1}{\sqrt{1-x^2}}\, dx = -\frac{1}{2}\int -2x\, (1-x^2)^{-\frac{1}{2}}\, dx = -\frac{1}{2}\cdot\frac{(1-x^2)^{-\frac{1}{2}+1 = \frac{1}{2}}}{\frac{1}{2}} =$

$$= -\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\sqrt{1-x^2} + c$$

INTEGRALI CICLICI

$$\rightarrow \underbrace{\int \underset{f'}{e^x} \cdot \underset{g}{\operatorname{sen} x}\, dx}_{} \overset{\times \text{ PARTI}}{=} e^x \operatorname{sen} x - \int \underset{f'}{e^x} \cos x\, dx \overset{\times \text{ PARTI}}{=}$$

$$= e^x \operatorname{sen} x - \left[e^x \cos x - \int e^x (-\operatorname{sen} x)\, dx \right] =$$

$$= e^x \operatorname{sen} x - \left[e^x \cos x + \int e^x \operatorname{sen} x\, dx \right] =$$

$$= \underline{e^x \operatorname{sen} x - e^x \cos x - \int e^x \operatorname{sen} x\, dx}$$

Quindi considero l'equazione

$$\int e^x \operatorname{sen} x\, dx = e^x \operatorname{sen} x - e^x \cos x - \int e^x \operatorname{sen} x\, dx$$

TRASPORTO

$$\frac{\cancel{2} \int e^x \operatorname{sen} x\, dx}{\cancel{2}} = \frac{e^x \operatorname{sen} x - e^x \cos x}{2} + c$$

ok

$$\rightarrow \underbrace{\int \underset{f'}{e^x} \cdot \underset{g}{\cos x}\, dx}_{} = e^x \cos x - \int e^x (-\operatorname{sen} x)\, dx$$

$$= e^x \cos x + \int \underset{f'}{e^x} \underset{g}{\operatorname{sen} x}\, dx = \text{ per parti ancora}$$

$$= \underline{e^x \cos x + e^x \operatorname{sen} x - \int e^x \cos x\, dx}$$

Ora considero l'equazione

$$\int e^x \cos x\, dx = e^x \cos x + e^x \operatorname{sen} x - \int e^x \cos x\, dx$$

TRASPORTO

$$\frac{\cancel{2} \int e^x \cos x\, dx}{\cancel{2}} = \frac{e^x \cos x + e^x \operatorname{sen} x}{2} + c$$

ok

$$\rightarrow \int sen^2 x \, dx = \int \underset{f'}{senx} \cdot senx \, dx = (-\cos x)\, senx - \int (-\cos x) \cos x \, dx =$$

$$= - senx \cos x + \int \underbrace{\cos^2 x}_{\substack{\parallel \\ 1 - sen^2 x}} dx =$$

$$= - senx \cos x + \int (1 - sen^2 x) \, dx =$$

$$= - senx \cos x + \int 1 dx - \int sen^2 x \, dx =$$

$$= - senx \cos x + x - \int sen^2 x \, dx$$

Quindi

$$\int sen^2 x \, dx = - senx \cos x + x - \underbrace{\int sen^2 x \, dx}$$

TRASPORTO

$$\frac{\cancel{2} \int sen^2 x \, dx}{\cancel{2}} = \frac{- senx \cos x + x}{2} + C$$

ok

$$\rightarrow \int \cos^2 x \, dx = \int (1 - sen^2 x) \, dx = \int 1 dx - \underbrace{\int sen^2 x \, dx}_{\text{ove uso il calcolo precedente}} =$$

$$= x - \left(\frac{- senx \cos x + x}{2} + c \right) =$$

$$= \frac{2x + senx \cos x - x}{2} + c$$

$$= \frac{x + senx \cos x}{2} + c$$

ok