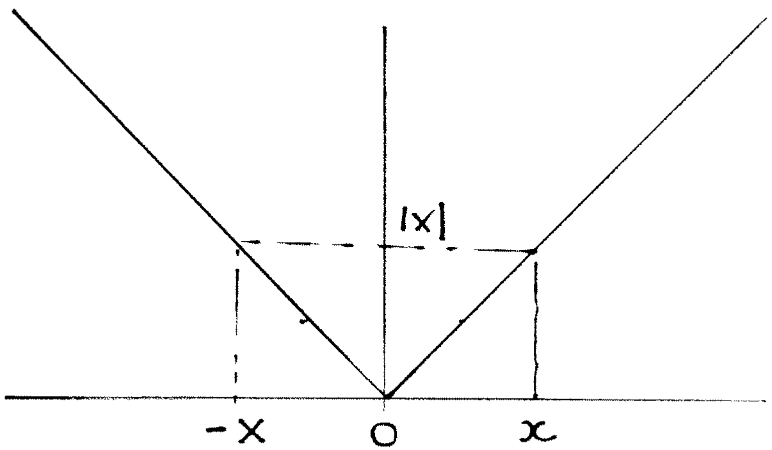
Funzione Modulo (Valore Assoluto)

$$f(x) = |x| = \begin{cases} x, & se\ x \geq 0 \\ -x, & se\ x < 0 \end{cases}$$



$f: \mathbb{R} \longrightarrow \mathbb{R}$

OSSERVAZIONE $\quad |x| \leq r \iff -r \leq x \leq r$

Proprietà

i) $|x| \geq 0 \quad \forall x \in \mathbb{R} \quad$ e $\quad |x| = 0 \iff x = 0$

ii) $|x_1 \cdot x_2| = |x_1| \cdot |x_2| \qquad \forall x_1, x_2 \in \mathbb{R}$

iii) $|x_1 + x_2| \leq |x_1| + |x_2| \qquad \forall x_1, x_2 \in \mathbb{R}$

Disuguaglianza Triangolare

$$|x_1 + x_2| \leq |x_1| + |x_2| \qquad \forall x_1, x_2 \in \mathbb{R}$$

<u>Dim</u>

$$-|x_1| \leq x_1 \leq |x_1| \qquad \forall x_1 \in \mathbb{R}$$

$$-|x_2| \leq x_2 \leq |x_2| \qquad \forall x_2 \in \mathbb{R}$$

$$\Longrightarrow$$

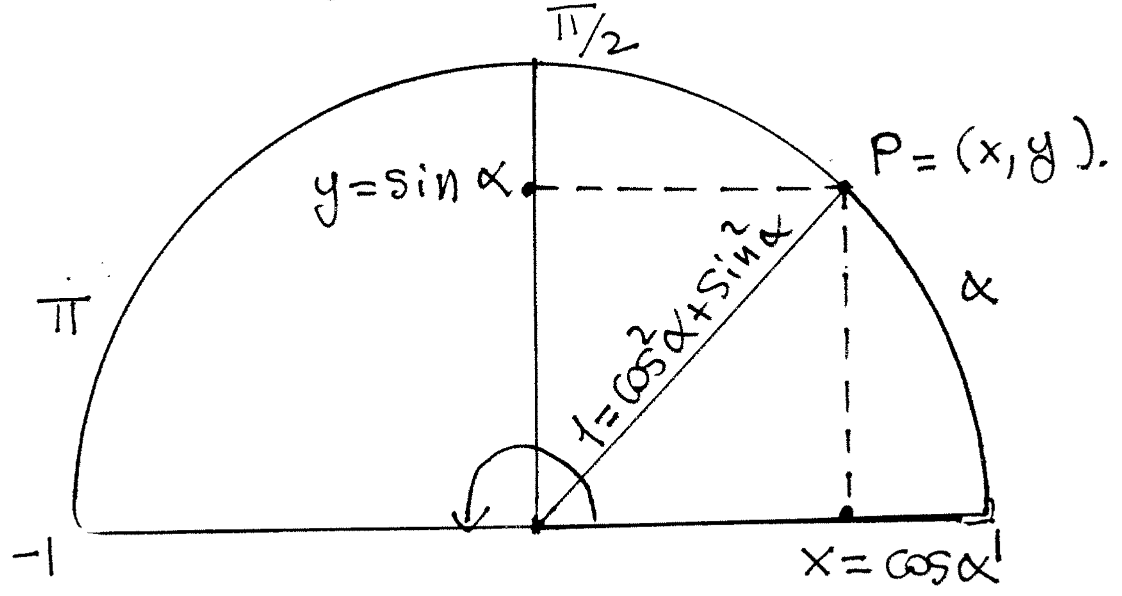
$$-|x_1| - |x_2| \leq x_1 + x_2 \leq |x_1| + |x_2| \qquad \forall x_1, x_2 \in \mathbb{R}$$

$$\Longleftrightarrow \quad |x_1 + x_2| \leq |x_1| + |x_2| \qquad \forall x_1, x_2 \in \mathbb{R}$$

Funzioni Trigonometriche

Unità di misura degli angoli = radianti

π = lungh. dell'arco semicrf di raggio 1



Un arco di lungh α individua un unico punto P sulla crf unitaria

$P=(x,y)$

Definiamo

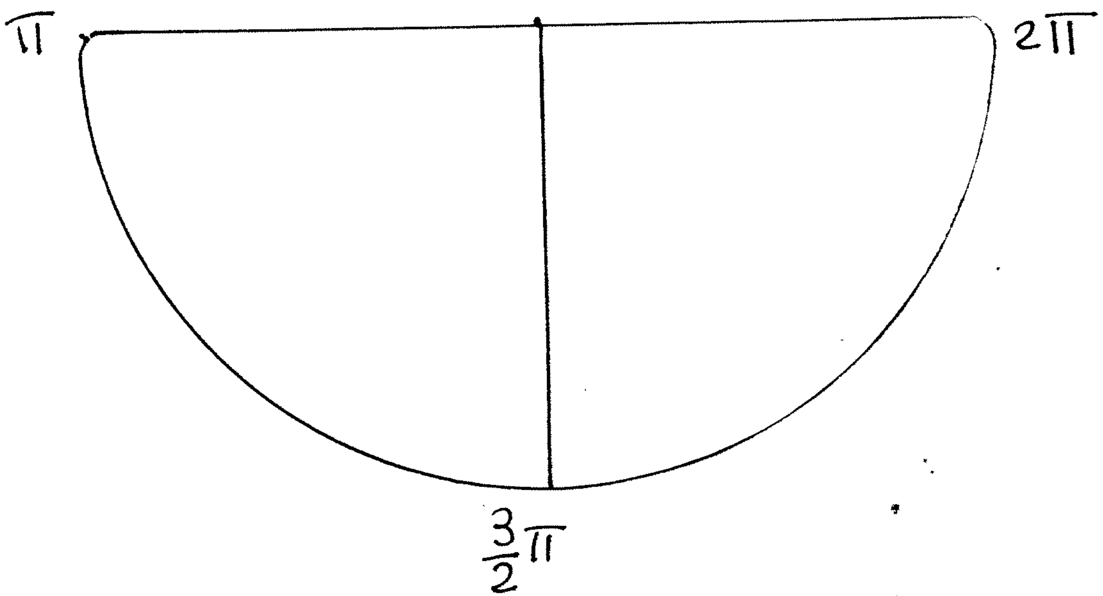
$x = \cos\alpha$

$y = \sin\alpha$

Esempio

$\alpha = \pi$ $\quad \cos\pi = -1$, $\quad \sin\pi = 0$

$$\boxed{\cos 0 = 1 \quad \sin 0 = 0}\ , \quad \boxed{\cos\pi/2 = 0 \quad \sin\pi/2 = 1}$$



$$\boxed{\begin{array}{l} \cos \frac{3}{2}\pi = 0 \\ \sin \frac{3}{2}\pi = -1 \end{array}}$$

$\cos \alpha = \cos(\alpha + 2k\pi) \quad \forall k \in \mathbb{Z}$

$\sin \alpha = \sin(\alpha + 2k\pi) \quad \forall k \in \mathbb{Z}$

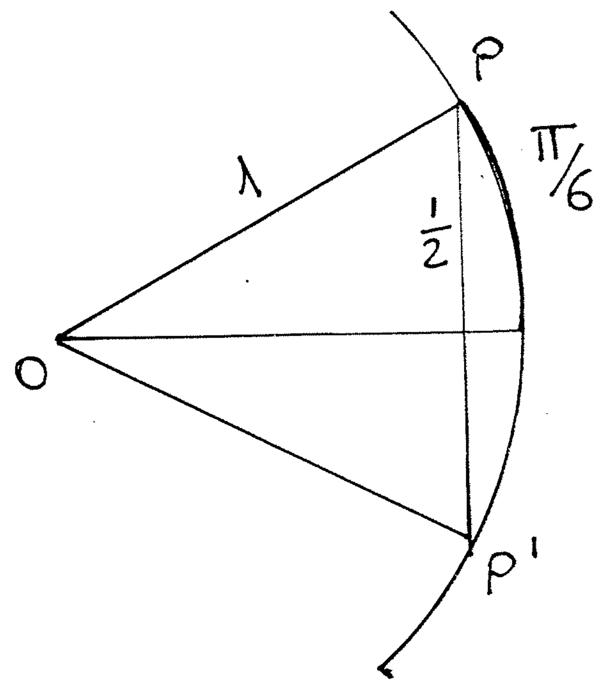
Funzioni 2π-periodiche

$\cos : \mathbb{R} \longrightarrow [-1, 1]$

$\sin : \mathbb{R} \longrightarrow [-1, 1]$

Si usa anche la notazione sen = sin

$\alpha = \frac{\pi}{6}$



Il triangolo OPP' è equilatero

$$\sin\frac{\pi}{6} = \frac{1}{2}$$

$$\cos\frac{\pi}{6} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$\alpha = \frac{\pi}{3}$ $\quad \cos\frac{\pi}{3} = \frac{1}{2}$, $\operatorname{sen}\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2}{3}\pi$ $\quad \cos\frac{2}{3}\pi = -\frac{1}{2}$, $\operatorname{sen}\frac{2}{3}\pi = \frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5}{6}\pi$ $\quad \cos\frac{5}{6}\pi = -\frac{\sqrt{3}}{2}$, $\operatorname{sen}\frac{5}{6}\pi = \frac{1}{2}$

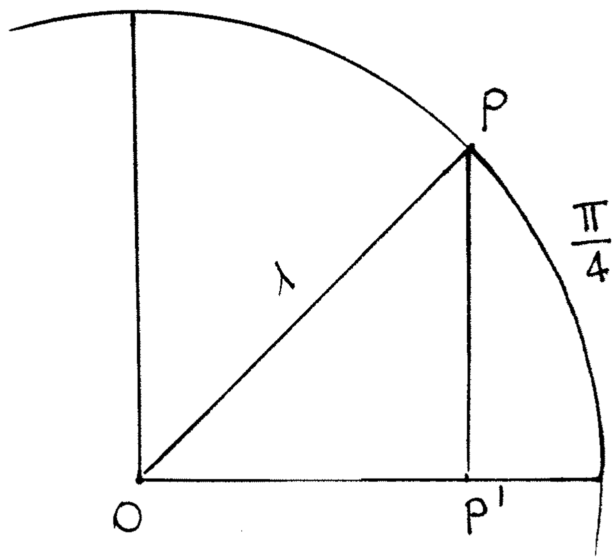
$\alpha = \pi + \frac{\pi}{6} = \frac{7}{6}\pi$ $\quad \cos\frac{7}{6}\pi = -\frac{\sqrt{3}}{2}$, $\operatorname{sen}\frac{7}{6}\pi = -\frac{1}{2}$

$\alpha = \pi + \frac{\pi}{3} = \frac{4}{3}\pi \qquad \cos\frac{4}{3}\pi = -\frac{1}{2} \ , \ \operatorname{sen}\frac{4}{3}\pi = -\frac{\sqrt{3}}{2}$

$\alpha = \frac{3}{2}\pi + \frac{\pi}{6} = \frac{5}{3}\pi \qquad \cos\frac{5}{3}\pi = \frac{1}{2} \ , \ \operatorname{sen}\frac{5}{3}\pi = -\frac{\sqrt{3}}{2}$

$\alpha = \frac{3}{2}\pi + \frac{\pi}{3} = \frac{11}{6}\pi \qquad \cos\frac{11}{6}\pi = \frac{\sqrt{3}}{2} \ , \ \operatorname{sen}\frac{11}{6}\pi = -\frac{1}{2}$

$\alpha = \frac{\pi}{4}$



Il triangolo rettangolo OP'P è isoscele

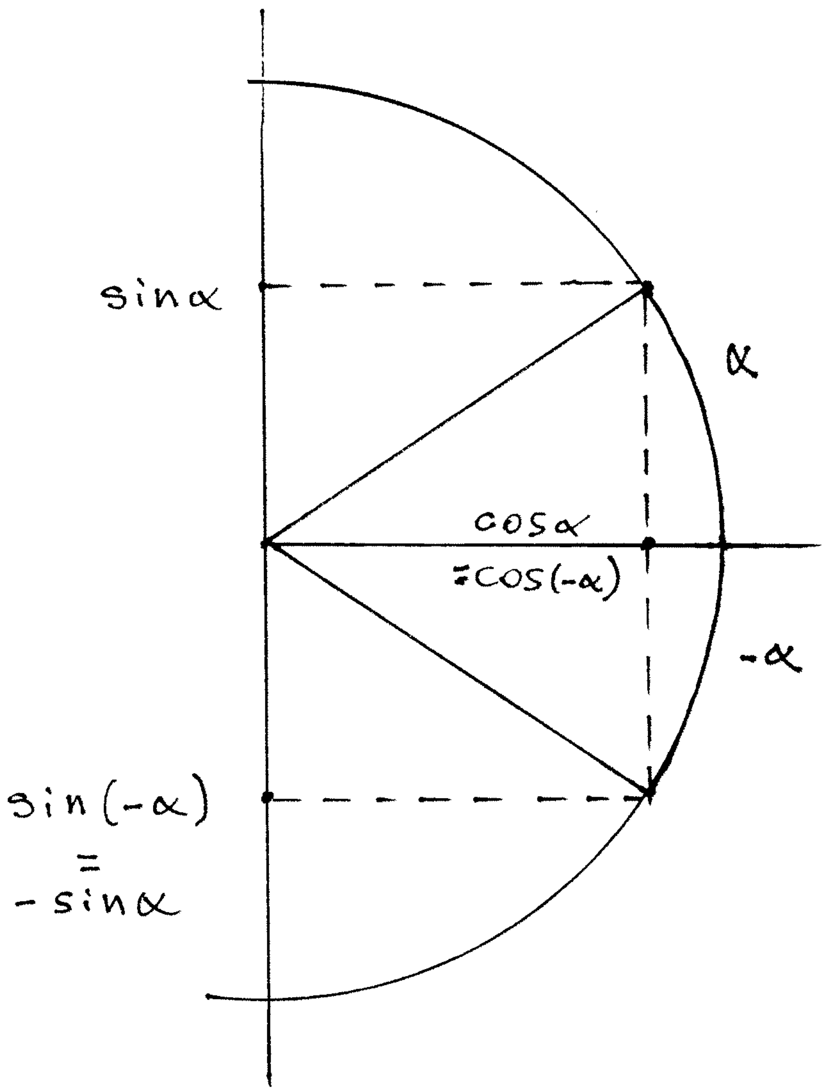
$$\begin{cases} \cos\frac{\pi}{4} = \sin\frac{\pi}{4} \geq 0 \\ \cos^2\frac{\pi}{4} + \sin^2\frac{\pi}{4} = 1 \end{cases}$$

$$\Rightarrow \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$\alpha = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3}{4}\pi$, $\cos\frac{3}{4}\pi = -\frac{1}{\sqrt{2}}$, $\sin\frac{3}{4}\pi = \frac{1}{\sqrt{2}}$

$\alpha = \pi + \frac{\pi}{4} = \frac{5}{4}\pi$, $\cos\frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$, $\sin\frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$

$\alpha = \frac{3}{2}\pi + \frac{\pi}{4} = \frac{7}{4}\pi$, $\cos\frac{7}{4}\pi = \frac{1}{\sqrt{2}}$, $\sin\frac{7}{4}\pi = -\frac{1}{\sqrt{2}}$

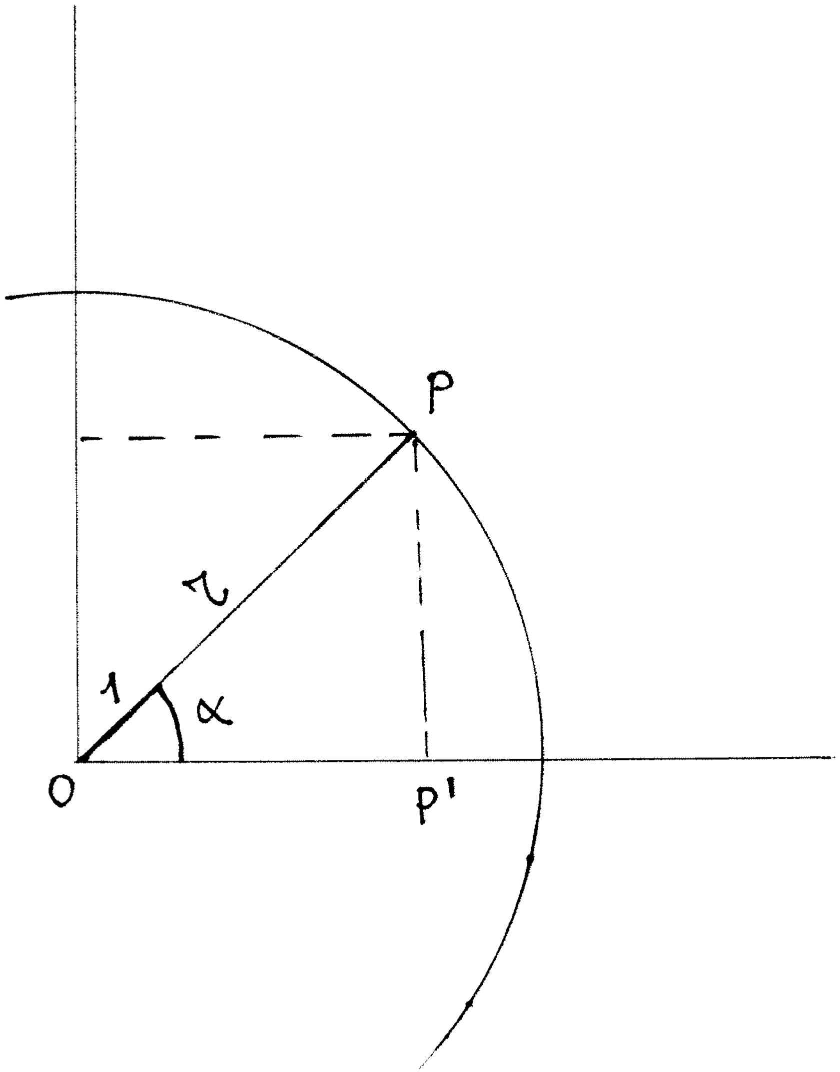


cos è una funzione pari

$$\cos(-\alpha) = \cos\alpha \quad \forall \alpha \in \mathbb{R}$$

sin è una funzione dispari

$$\sin(-\alpha) = -\sin\alpha \quad \forall \alpha \in \mathbb{R}$$



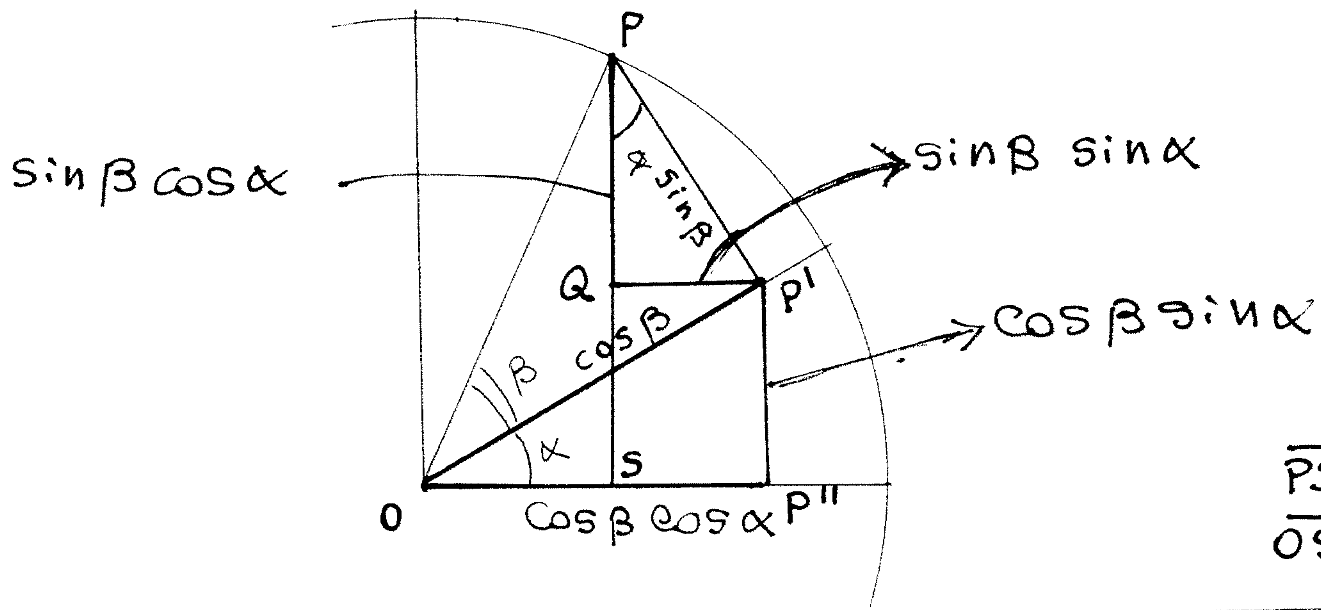
$r > 0$

$\overline{OP} = r$

$\overline{OP'} = r\cos\alpha$

$\overline{PP'} = r\sin\alpha$

Formule di Addizione



$\overline{OP} = 1$

$\overline{OP'} = \cos\beta$

$\overline{PP'} = \sin\beta$

$\overline{OP''} = \cos\beta \cos\alpha$

$\overline{P'P''} = \cos\beta \sin\alpha$

$\overline{PQ} = \sin\beta \cos\alpha$

$\overline{QP'} = \sin\beta \sin\alpha$

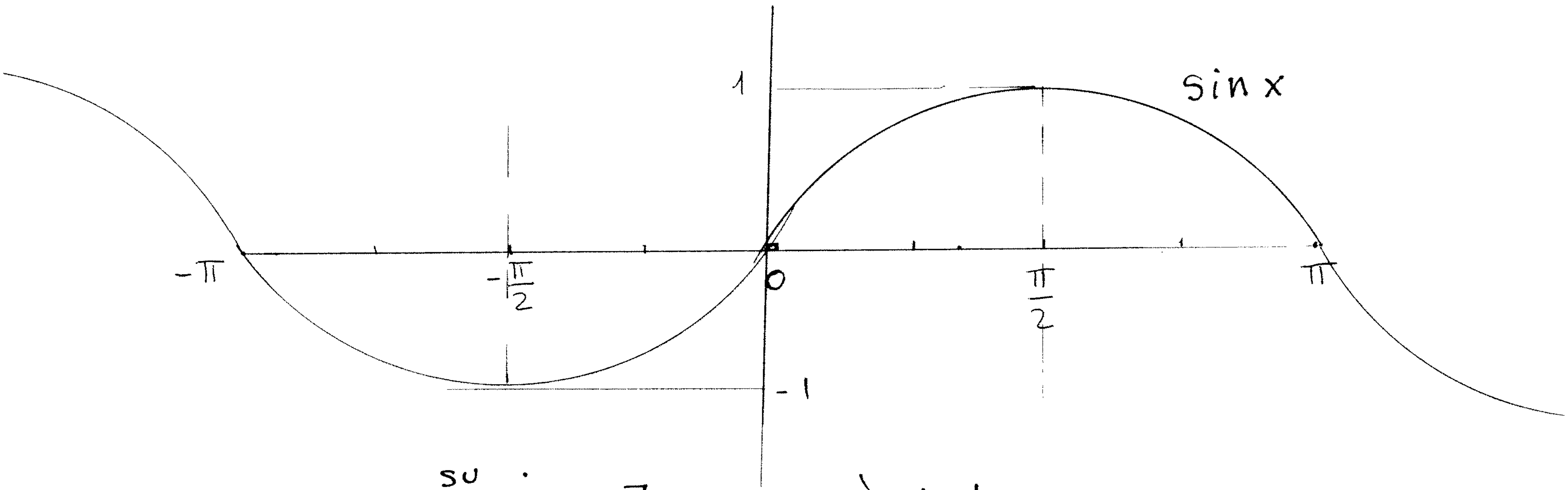
$\overline{PS} = \overline{PQ} + \overline{P'P''} = \sin(\alpha+\beta)$

$\overline{OS} = \overline{OP''} - \overline{QP'} = \cos(\alpha+\beta)$

$$\boxed{\begin{aligned} \cos(\alpha+\beta) &= \cos\beta \cos\alpha - \sin\beta \sin\alpha \\ \sin(\alpha+\beta) &= \sin\alpha \cos\beta + \sin\beta \cos\alpha \end{aligned}}$$

$$\cos(\alpha-\beta) = \cos(-\beta)\cos\alpha - \sin(-\beta)\sin\alpha$$
$$= \cos\beta \cos\alpha + \sin\beta \sin\alpha$$

$$\sin(\alpha-\beta) = \sin\alpha \cos(-\beta) + \sin(-\beta)\cos\alpha$$
$$= \sin\alpha \cdot \cos\beta - \sin\beta \cos\alpha$$

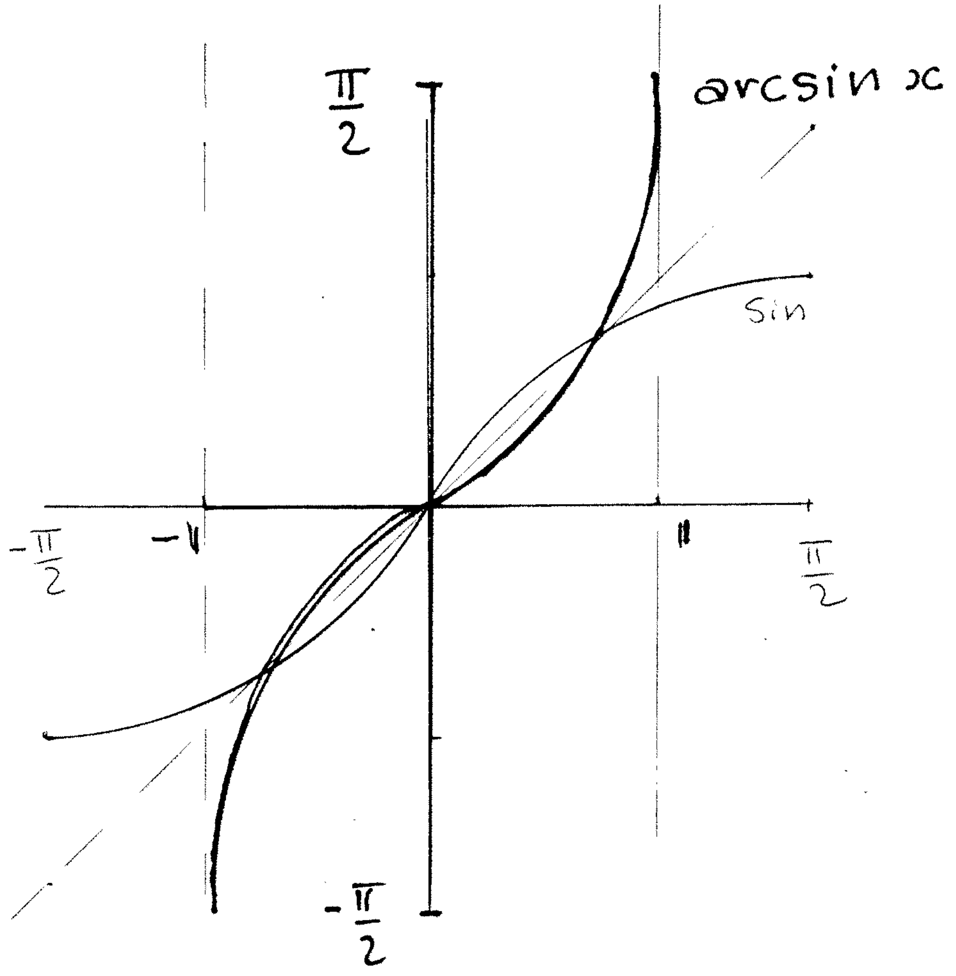


$\sin : \mathbb{R} \xrightarrow{su} [-1, 1]$ non è 1-1

$\sin : [-\frac{\pi}{2}, \frac{\pi}{2}] \xrightarrow[1-1]{su} [-1, 1]$ $\uparrow$ strettamente

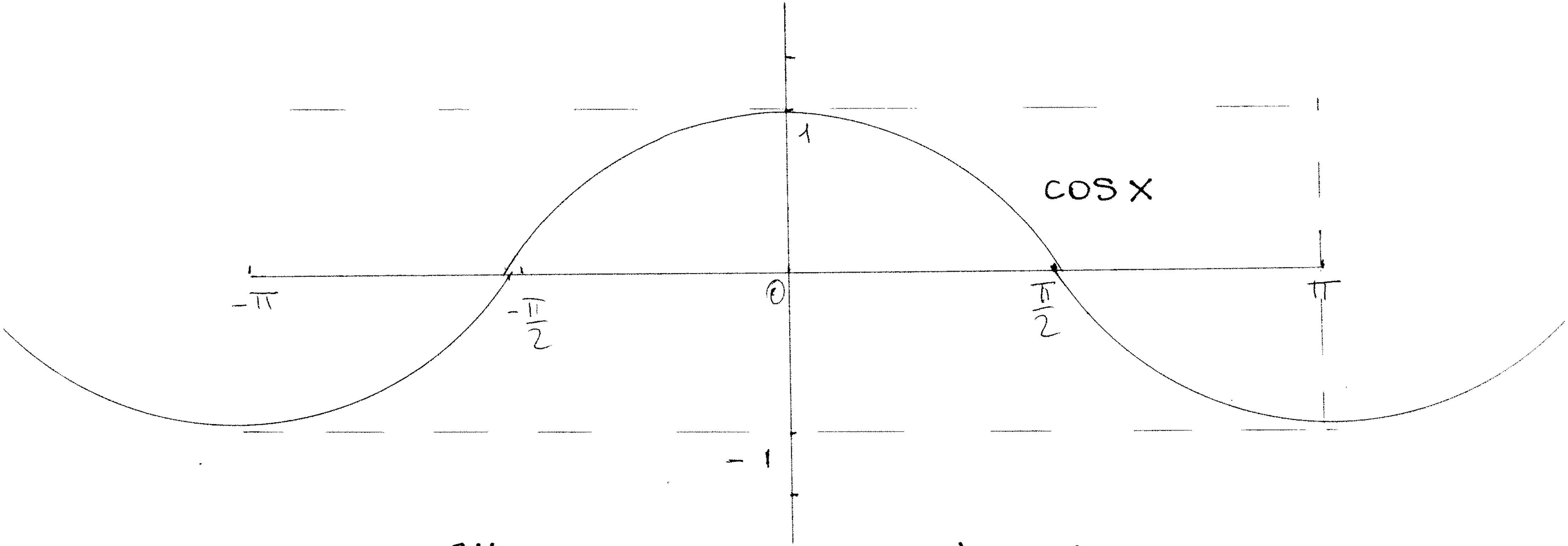
La sua funzione inversa è

$\arcsin : [-1, 1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$



$$\arcsin : [-1, 1] \xrightarrow[\text{su}]{1-1} \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$\uparrow$

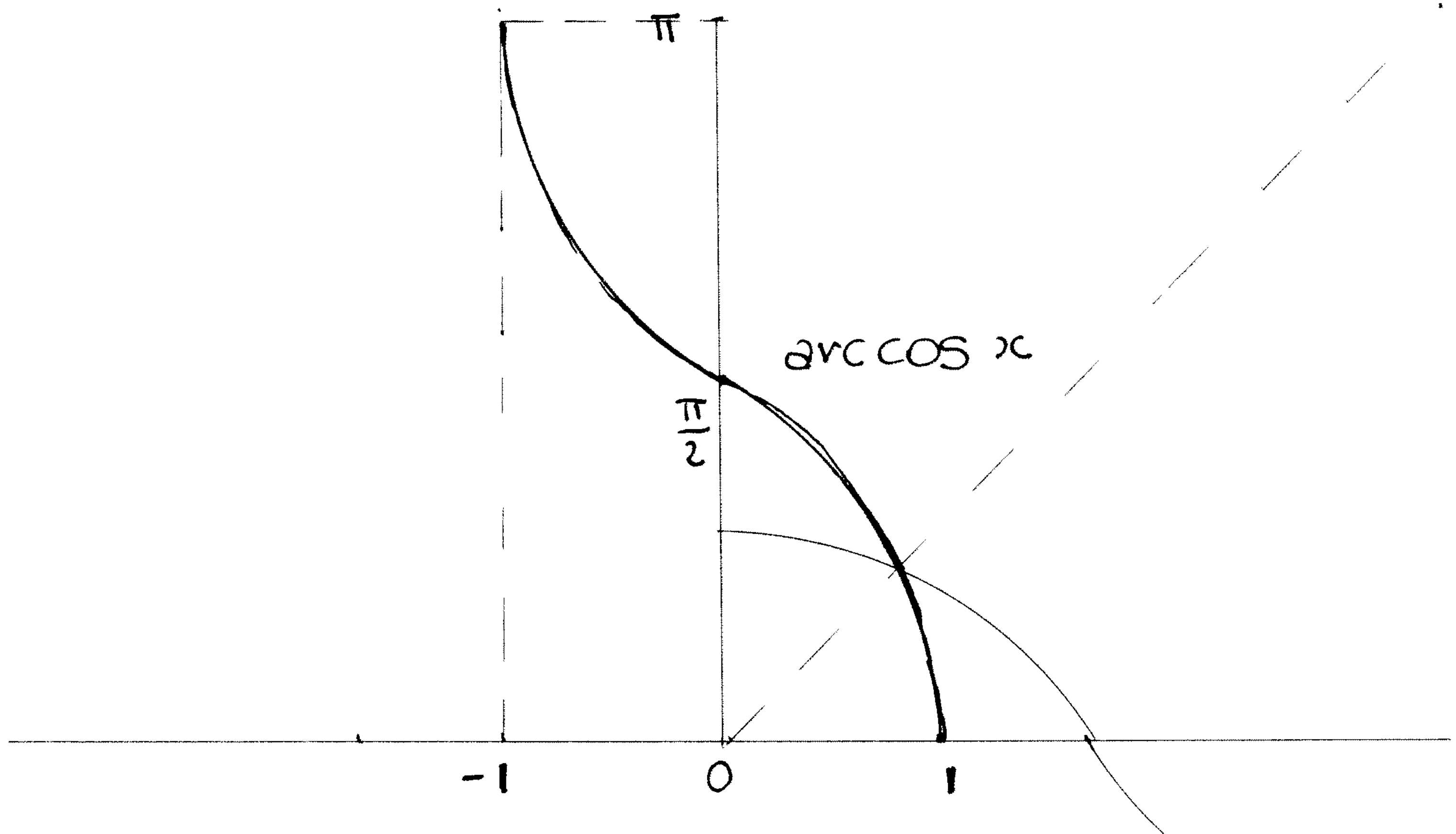


$\cos : \mathbb{R} \xrightarrow{su} [-1, 1]$ non e' 1-1

$\cos : [0, \pi] \xrightarrow[su]{1-1} [-1, 1]$ $\downarrow$ strettamente

La sua funzione inversa e'

$\arccos : [-1, 1] \longrightarrow [0, \pi]$



$\arccos : [-1, 1] \longrightarrow [0, \pi]$