Exercises
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Exercise 3:

a) Show: Two Bézier curves

$$\overrightarrow{x_l}(u) = \sum_{i=0}^{n} \overrightarrow{b_{nl+i}} B_i^n\left(\frac{u-u_l}{u_{l+1}-u_l}\right), u \in [u_l, u_{l+1}]$$

$$\overrightarrow{x_{l+1}}(u) = \sum_{i=0}^{n} \overrightarrow{b_{n(l+1)+i}} B_i^n\left(\frac{u-u_{l+1}}{u_{l+2}-u_{l+1}}\right), u \in [u_{l+1}, u_{l+2}]$$

that are C^2–continuous in their common point $\overrightarrow{x_l}(u_{l+1}) = \overrightarrow{x_{l+1}}(u_{l+1})$ have an equidistant parameterisation if and only if

$$\overrightarrow{b_{n(l+1)+2}} - \overrightarrow{b_{n(l+1)-2}} \parallel \overrightarrow{b_{n(l+1)+1}} - \overrightarrow{b_{n(l+1)-1}}$$

b) The curvature $\kappa(u)$ of a parametric curve $\overrightarrow{x}(u)$ in E^2 is given by

$$\kappa(u) = \frac{det(\dot{\overrightarrow{x}}(u), \ddot{\overrightarrow{x}}(u))}{\|\dot{\overrightarrow{x}}(u)\|^3}$$

As in a), consider two Bézier segments in E^2 that are equidistantly parametrized and C^2 continuous in $\overrightarrow{b_0}$, where $n = 3$ et $l = -1$.

Prove: Despite moving the Bézier point $\overrightarrow{b_2}$ along the line through the points $\overrightarrow{b_{-2}}$ and $\overrightarrow{b_2}$ the Bézier segments always join with curvature continuity in $\overrightarrow{b_0}$.

Exercise 4:
Let $P_0(0,0), P_1(1,0), P_2(0,1)$ be the vertices of the triangle Δ. Construct a Bézier curve with the following properties:

- c is closed and lies completely in the interior of Δ.
- c is composed by three cubic Bézier curves that join with C^2 continuity. (Control points respectively: $\overrightarrow{b_0}, \overrightarrow{b_1}, \overrightarrow{b_2}, \overrightarrow{b_3}$; $\overrightarrow{b_3}, \overrightarrow{b_4}, \overrightarrow{b_5}, \overrightarrow{b_6}$; $\overrightarrow{b_6}, \overrightarrow{b_7}, \overrightarrow{b_8}, \overrightarrow{b_9} = \overrightarrow{b_0}$.)
- c is tangent to each of the triangle edges (contact points $\overrightarrow{b_0} = \overrightarrow{b_9} \in P_0P_1, \overrightarrow{b_3} \in P_1P_2, \overrightarrow{b_6} \in P_2P_0$).
- c has an equidistant parameterisation.

The control point $\overrightarrow{b_1} = (\frac{2}{3}, 0)^T$ is already determined.