Progetto 19 ODE
Newton's Law of Cooling

A cup of coffee cooling on the counter, a cake warming in the oven, and a body found in the chill autumn weather . . . are these the ingredients for a murder mystery to read by the fire or a case for that most famous detective of natural phenomena, Sir Isaac Newton? We use a first-order linear differential equation formulated by Newton to predict the temperatures of objects introduced into media with known ambient temperatures.

1. A Basic Differential Model

A simple method to model the cooling or heating of an object placed in a constant ambient temperature is to say that the time rate of change in temperature is proportional to the difference between the temperature *A* of the surrounding medium (the ambient temperature) and the temperature *T* of the object:

$$\frac{\mathrm{dT}}{\mathrm{dx}} = k\big(A - T(t)\big) \qquad k > 0$$

The general solution for Equation (1) is $T=A+Ce^{-kT}$. If the ambient temperature *A* is given, then two data points are required to determine the constants *C* and *k*. This equation is called **Newton's Law of Cooling and Heating.** In this model Newton assumed that the heat transfer between the object and the surrounding environment is not sufficient to affect noticeably the ambient temperature.

1.1 If $T > A$, is the object cooling or heating? Is $\frac{\mathrm{dT}}{\mathrm{dx}}$ positive, negative, or zero when $T > A$?

1.2 For what value of *T* is $\frac{\mathrm{dT}}{\mathrm{dx}} = 0$? This value is called the **equilibrium** value.

1.3 If $T(0) = T0$, solve equation (1) analytically to show that

$$\mathrm{T} = \mathrm{A} + (\mathrm{T0} - \mathrm{A})\mathrm{e}^{-\mathrm{kT}}$$

Experiment with various values for the constant of proportionality *k* and the ambient temperature *A*.

2.1 What do you notice about the behavior of the curve on the vs. *t* graph (in the upper-right corner)? What happens to as *t* becomes large? Vary *k* and *A*. Is the long-term behavior of always the same?

2.2 Look at the graph of $A - T$ vs.T . How do you interpret the straight-line graph? What does the slope of the line denote?

Coroners use several methods to determine time of death. If Equation (1) were used, measurements of the temperature at two different times would be required to establish *k* and the constant of integration. Suppose this were the only method used to determine time of death in a case where the time of death was the crucial element in the prosecution's case. How would you, as the scientific consultant, help the defense cast doubt on this estimate? Think carefully about the assumptions of the model!