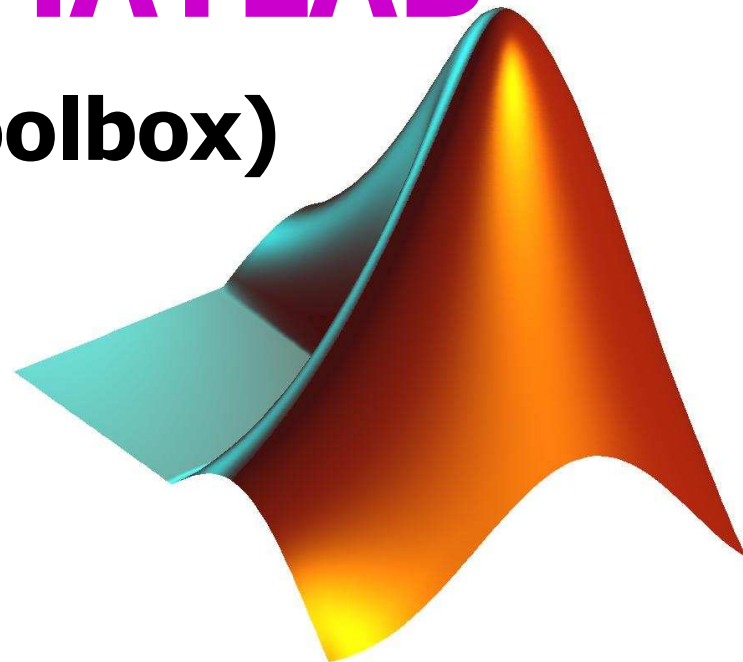


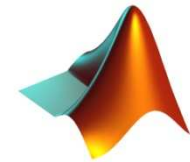
Introduzione al calcolo simbolico in MATLAB

(Symbolic math toolbox)



Analisi Numerica - A.A. 2010/11

Calcolo Simbolico in MATLAB



Fino ad ora si è utilizzato MATLAB per eseguire solo operazioni numeriche. In realtà spesso è utile manipolare espressioni matematiche con l'ausilio del calcolatore per ottenere risultati in forma analitica.

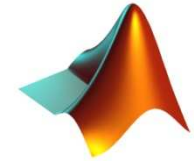
Es: Consideriamo l'equazione $y = \frac{2(x+3)^2}{x^2 + 6x + 9}$

Sfruttando il calcolo simbolico,
essa è equivalente a

$$y = \frac{2(x+3)^2}{x^2 + 6x + 9} = \frac{2 \cdot (x^2 + 6x + 9)}{(x^2 + 6x + 9)} = 2$$

In Matlab variabili ed espressioni di tipo simbolico possono essere definite e trattate sfruttando le potenzialità del **Symbolic math toolbox**

Help e Demos



Il **Symbolic Math Toolbox** utilizza molti dei nomi delle funzioni numeriche di MATLAB e per ottenere le informazioni relative alla versione simbolica di una particolare funzione occorre digitare nella Command Window

>>help *syml/nomefunzione*

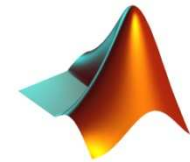
Può essere utile anche consultare le dimostrazioni e gli esempi contenuti nel MATLAB Demos. Digitare quindi

>>demos

e cliccare su

Toolboxes → **Symbolic Math**

Le variabili simboliche



Il **Symbolic Math Toolbox** definisce un nuovo tipo di variabile, chiamato **oggetto simbolico**. E' una struttura dati che memorizza una rappresentazione stringa del simbolo. Per creare oggetti simbolici in MATLAB si utilizza la funzione **sym**.

Esempio:

```
>> x=sym('x')  
( >> syms x )  
x =  
x
```

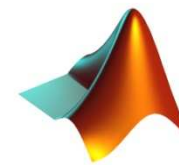
```
>> class(x)  
ans =  
sym
```

```
>> t=6;  
>> g=sym(t)  
g =  
6
```

```
>> class(g)  
ans =  
sym
```

```
>> class(t)  
ans =  
double
```

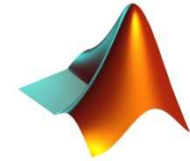
Rappresentazione simbolica di un valore numerico



>>t=0.1;

sym(t,'r')	rappresentazione razionale (default)	1/10
sym(t,'f')	rappresentazione floating-point	(2 ⁻⁴ +2702159776422298*2 ⁻⁵⁶)
sym(t,'d')	espansione decimale con 32 cifre significative	.10000000000000000000055511512312578
digits(7) sym(t,'d')	espansione decimale con 7 cifre significative	.1000000

Creare funzioni matematiche simboliche



```
>>syms x y z real
```

VARIABILI SIMBOLICHE
REALI

```
>>r = sqrt(x^2 + y^2 + z^2)
```

```
r =
```

```
(x^2+y^2+z^2)^(1/2)
```

ESEMPI DI ESPRESSIONI
SIMBOLICHE

```
>>t = atan(y/x)
```

```
t =
```

```
atan(y/x)
```

```
>> f=r+t
```

OPERAZIONI TRA ESPRESSIONI SIMBOLICHE

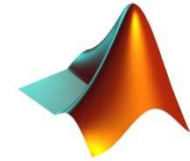
```
f =
```

```
(x^2+y^2+z^2)^(1/2)+atan(y/x)
```

Funzioni per manipolare espressioni simboliche

collect(E)	raccoglie i coefficienti con la stessa potenza di x
expand(E)	applica regole algebriche per espandere l'espressione E
factor(E)	esprime E come prodotto di polinomi con coefficienti razionali
poly2sym(p)	converte i coefficienti del vettore p in un polinomio simbolico
sym2poly(E)	converte l'espressione E nel vettore di coefficienti
pretty(E)	visualizza l'espressione E in forma matematica
simple(E)	ricerca la forma dell'espressione E più corta in termini di numero di caratteri, utilizzando differenti semplificazioni algebriche
simplify(E)	semplifica l'espressione E
subs(E,old,new)	sostituisce <i>new</i> al posto di <i>old</i> nell'espressione E

Esempi



```
1) >> x=sym('x');  
>> E=(x-1)*(x-2)*(x-3);  
>> collect(E)  
ans =  
x^3-6*x^2+11*x-6
```

```
2) >> E=(x-5)^2+(y-3)^2;  
>> collect(E,y)  
ans =  
y^2-6*y+9+ (x-5)^2
```

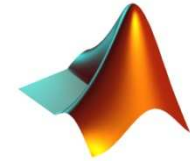
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3) >> E=cos(x+y);  
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cos(x)*cos(y)-sin(x)*sin(y)
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4) >> E= x^3-6*x^2+11*x-6;  
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(x-1)*(x-2)*(x-3)
```

```
5) >> p=[2 6 4];  
>> poly2sym(p)  
ans =  
2*x^2+6*x+4
```

```
6) >> E=5*y^2-3*y+7  
>> sym2poly(E)  
ans =  
[5 -3 7]
```

Esempi



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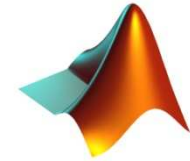
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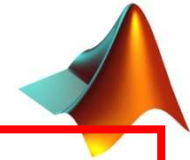
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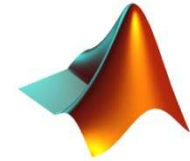
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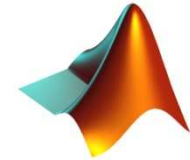
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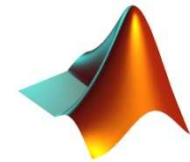
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Esempi



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>> pretty(E)
ans=

$$x^3 - 6x^2 + 11x - 6$$

```

```
2) >> E=(1-x^2)/(1-x)
>> simplify(E)
ans=

$$x+1$$

```

```
3) >> E = cos(x)^2 + sin(x)^2
>> simplify(E)
ans=

$$1$$

```

```
4) >> E = x^2+6*x+7
>> subs(E,x,2)
ans=

$$23$$

```

```
5) >> E = a*sin(b)
>> subs(E, {a,b}, {x,2})
ans=

$$x*\sin(2)$$

```

```
6) >> E = 3*cos(x)^2+sin(x)^2
>> simplify(E)
ans=

$$2*\cos(x)^2+1$$

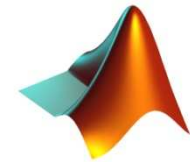
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```
7) >> E = 3*cos(x)^2+sin(x)^2
>> simple(E)
ans=

$$\cos(2*x)+2$$

```

Esempi



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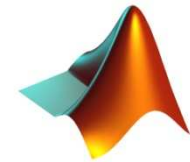
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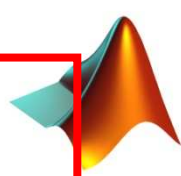
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Esempi

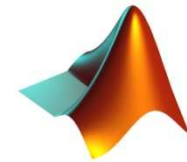
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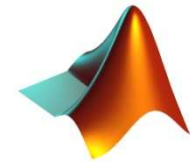
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Esempi



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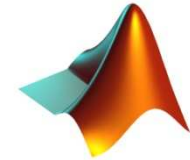
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Esempi



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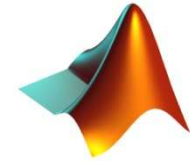
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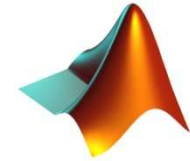
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>> simple(E)
ans=
cos(2*x)+2
```

Funzioni per creare e valutare espressioni simboliche



class(E)	restituisce la classe dell'espressione E
double(E)	converte l'espressione E in forma numerica
findsym(E)	restituisce il nome delle variabili contenute in E
[num,den]=numden(E)	restituisce due espressioni simboliche che rappresentano il numeratore e il denominatore della rappresentazione razionale di E
vpa(E,d)	usa l'aritmetica a precisione variabile per calcolare gli elementi di E con d cifre decimali

Esempi



```
1) >>syms x
>>E=(x-1)*(x-2)*(x-3)
>>class(E)
ans=
sym
```

```
2) >>E=sym('(1+sqrt(5))/2')
>>double(E)
ans=
1.6180
```

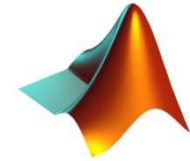
```
3) >>digits(25)
>>vpa(pi)
ans=
3.141592653589793238462643
```

```
4) >>E= x/y + y/x
>> [num den]=numden(E)
num=x^2+y^2
den=y*x
```

```
5) >>E=x+i*y-j*z
>> findsym(E)
ans=
x, y, z
```

```
6) >>E= x^2-6*x+7
>>ezplot(E,[-2 6])
```

Esempi



1) `>>syms x`

`>>E=(x-1)*(x-2)*(x-3)`

`>>class(E)`

ans=

sym

2) `>>E=sym('(1+sqrt(5))/2')`

`>>double(E)`

ans=

1.6180

3) `>>digits(25)`

`>>vpa(pi)`

ans=

3.141592653589793238462643

4) `>>E= x/y + y/x`

`>> [num den]=numden(E)`

num=x^2+y^2

den=y*x

5) `>>E=x+i*y-j*z`

`>> findsym(E)`

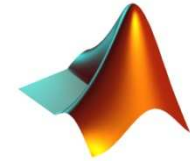
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Esempi



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>>E=(x-1)*(x-2)*(x-3)
>>class(E)
ans=
sym
```

```
2) >>E=sym('(1+sqrt(5))/2')
>>double(E)
ans=
1.6180
```

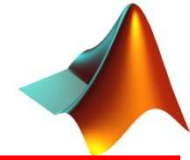
```
3) >>digits(25)
>>vpa(pi)
ans=
3.141592653589793238462643
```

```
4) >>E= x/y + y/x
>> [num den]=numden(E)
num=x^2+y^2
den=y*x
```

```
5) >>E=x+i*y-j*z
>> findsym(E)
ans=
x, y, z
```

```
6) >>E= x^2-6*x+7
>>ezplot(E,[-2 6])
```

Esempi



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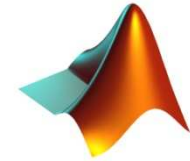
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Esempi



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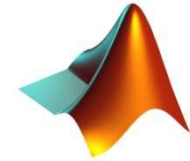
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Esempi



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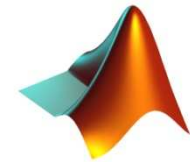
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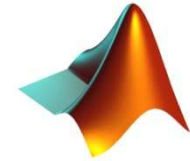
`>>ezplot(E,[-2 6])`

Funzioni per risolvere equazioni algebriche e trascendenti



solve(E)	Risolve una equazione oppure una espressione ($E=0$) simbolica. <i>Non è necessario dichiarare le variabili con sym o syms</i>
solve(E1, ..., En)	Risolve un sistema di equazioni o espressioni simboliche
S=solve(E)	Memorizza la soluzione in una struttura

Esempi



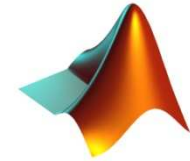
```
1) >>solve('x+5')  
ans =  
    -5
```

```
2) >>eq='exp(2*x)+3*exp(x)=54';  
>>solve(eq)  
ans =  
    [ log(-9)]  
    [ log(6)]
```

```
3) >>eq1= '6*x+2*y=14';  
>>eq2= '3*x+7*y=31';  
>>S=solve(eq1,eq2)  
S =  
    x: [1x1 sym]  
    y: [1x1 sym]
```

```
>>S.x  
ans =  
    1  
>>S.y  
ans =  
    4
```

Esempi



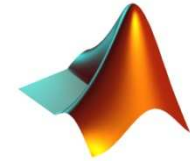
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ans =
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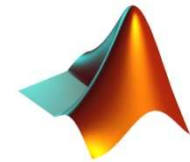
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```

```
>>S.x  
ans =  
    1  
  
>>S.y  
ans =  
    4
```

Esempi



```
4) >> syms a u v
>> f1=a*u^2+v^2;
>> f2=u-v-1;
>> f3=a^2-5*a+6;
>> A=solve(f1,f2,f3)
```

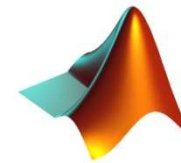
A =

a: [4x1 sym]

u: [4x1 sym]

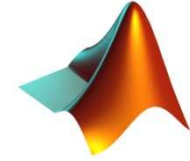
v: [4x1 sym]

Funzioni per il calcolo simbolico



diff(E)	Restituisce la derivata dell'espressione E rispetto alla variabile indipendente di default (x)
int(E)	Restituisce l'integrale dell'espressione E
limit(E)	Restituisce il valore del limite di E per x che tende a 0 (default)
symsum(E)	Restituisce la somma dell'espressione E rispetto alla sua variabile k da 0 a k-1
taylor(f,n,a)	Restituisce il polinomio di Maclaurin di f di ordine n-1, valutato nel punto x=a

Esempi



```
1) >>E= '(sin(x))^2';  
>>diff(E) [ >>diff('(sin(x))^2') ]  
ans =  
2*sin(x)*cos(x)
```

```
2) >> syms x y  
>> diff(x*sin(x*y),y)  
ans =  
x^2*cos(x*y)
```

```
3) >> syms x  
>> diff(x^3,2)  
ans =  
6*x
```

```
4) >> syms x y  
>>diff(x*sin(x*y),y,2)  
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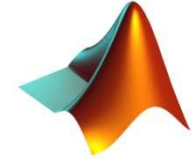
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ans =  
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```

```
6) >> syms x  
>> int(x^2,2,5)  
ans =  
39
```

```
7) >> syms x y  
>> int(x*y^2,y,0,5)  
ans =  
125/3*x
```

```
8) >> syms t x  
>> int(sin(x),t,exp(t))  
ans =  
-cos(exp(t))+cos(t)
```

Esempi



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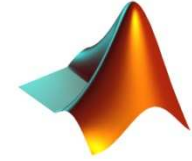
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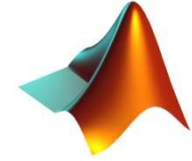
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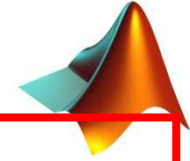
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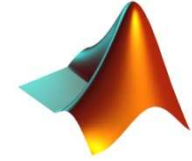
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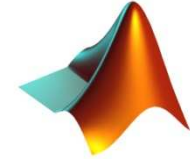
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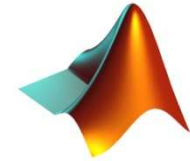
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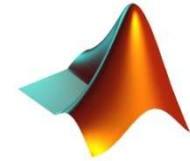
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```

Esempi



```
1) >>syms a x
>>limit(sin(a*x)/x)
ans =
a
```

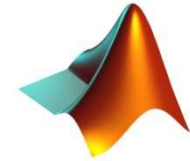
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3) >> syms x
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ans =
-inf
```

```
4) >> syms k x
>>symsum(k^2,1,4)
ans =
30
```

```
5) >> syms x
>> f=exp(x);
>>taylor(f,4)
ans =
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Esempi



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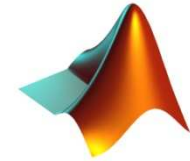
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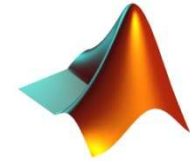
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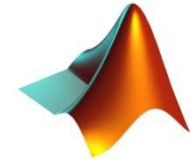
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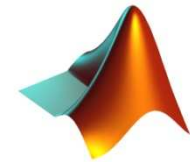
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```

ESERCIZIO

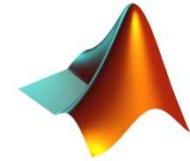


Supponete che dell'acqua venga pompata in un serbatoio inizialmente vuoto. E' noto che la velocità del flusso dell'acqua nel serbatoio al tempo t sia uguale a $50-t$ l/sec. Si può dimostrare che la quantità Q di acqua che fluisce nel serbatoio durante i primi x secondi è

$$\int_0^x 50 - t \, dt$$

- (a) determinare l'equazione simbolica che rappresenta la quantità di acqua nel serbatoio dopo x sec;
- (b) determinare la quantità di acqua nel serbatoio dopo 30 sec;
- (c) determinare la quantità di acqua che è fluita nel serbatoio tra 10 e 15 sec dopo l'inizio del flusso.

ESERCIZIO



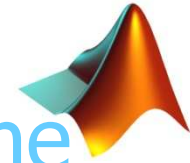
Supponete che il seguente polinomio rappresenti l'altitudine in metri durante le prime 48 ore dopo il lancio di un pallone meteorologico:

$$h(t) = -0.12t^4 + 12t^3 - 380t^2 + 4100t + 220$$

Le unità di t sono le ore.

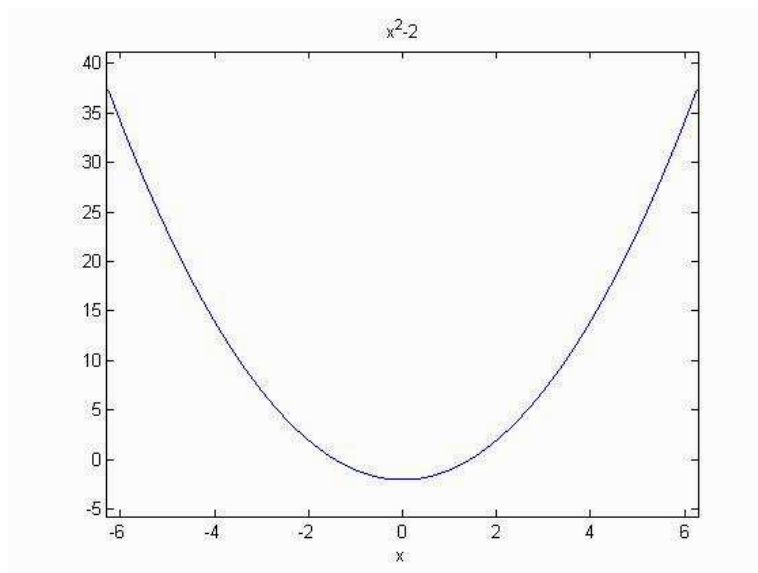
- (a) Utilizzare Matlab per determinare l'equazione per la velocità di salita oppure discesa del pallone.
- (b) Determinare l'equazione per l'accelerazione del pallone.
- (c) Disegnare i grafici della quota, della velocità e dell'accelerazione nell'intervallo $[0, 48]$.

Plot di variabili ed espressioni simboliche

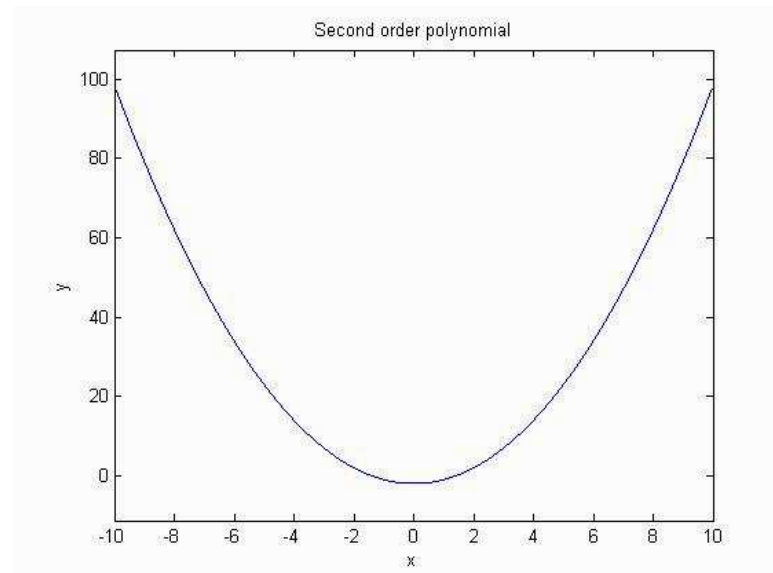


```
1) >>y=sym('x^2-2');  
>>ezplot(y);
```

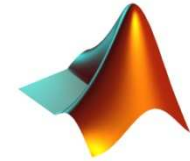
La funzione è valutata in $[-2\pi, 2\pi]$



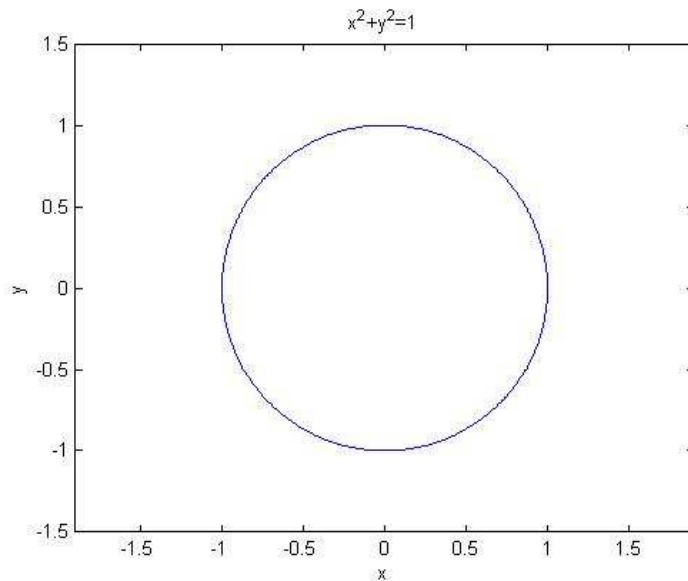
```
2) >>ezplot(y,[-10,10]);  
title('Second order polynomial')  
xlabel('x');  
ylabel('y');
```



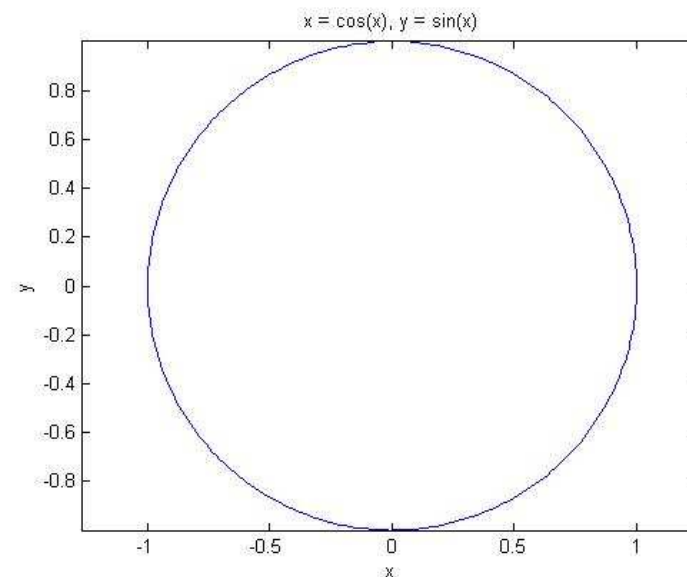
Grafici di funzioni implicite e parametriche



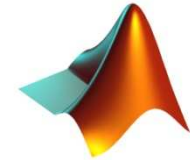
1) `>>ezplot('x^2+y^2=1',[-1.5,1.5])`
`>>ezplot('x^2+y^2-1',[-1.5,1.5])`
`>>z=sym('x^2+y^2-1')`
`>>ezplot(z, [-1.5,1.5])`



2) `>>ezplot('sin(x)','cos(x)');`

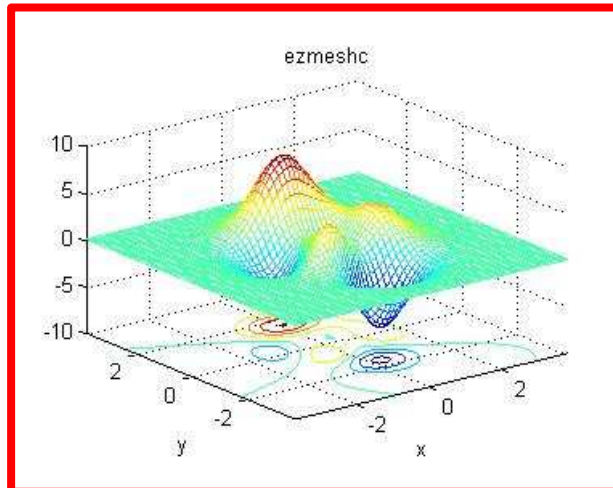


Grafici 3d

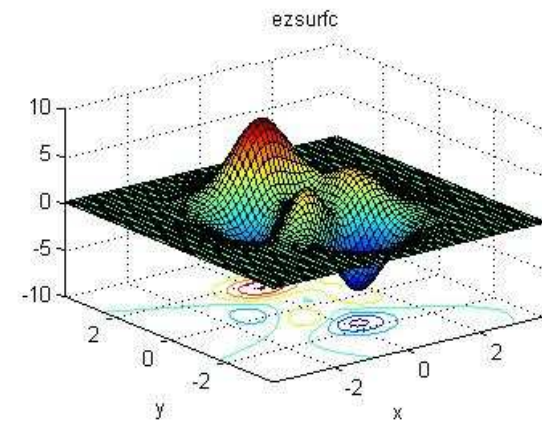


```
>>z1=sym('3*(1-x)^2*exp(-(x^2)-(y+1)^2)');  
>>z2=sym('-10*(x/5-x^3-y^5)*exp(-x^2-y^2)');  
>>z3=sym('-1/3*exp(-(x+1)^2-y^2)');  
>>z=z1+z2+z3;
```

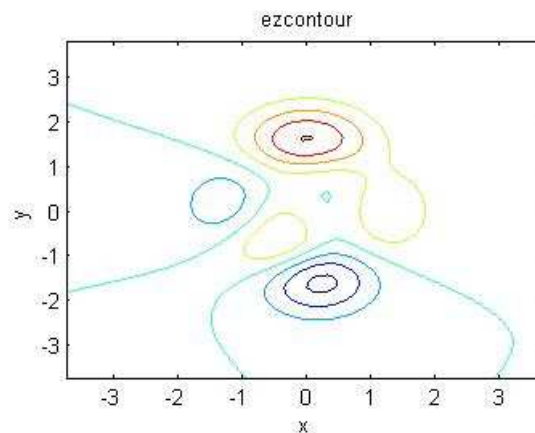
```
>>subplot(2,2,1);  
>>ezmeshc(z);  
>>title('ezmeshc');
```



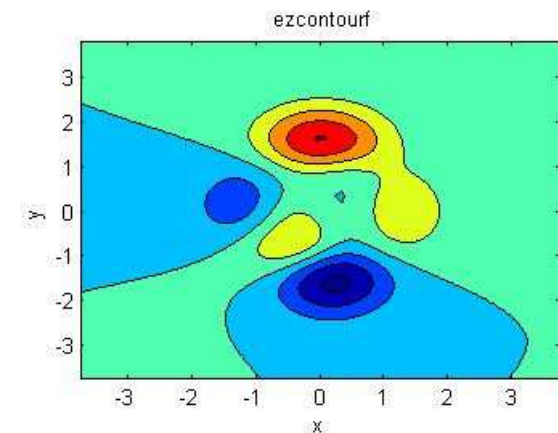
```
>>subplot(2,2,2);  
>>ezsurf(z);  
>>title('ezsurf');
```



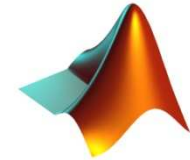
```
>>subplot(2,2,3);  
>>ezcontour(z);  
>>title('ezcontour');
```



```
>>subplot(2,2,4);  
>>ezcountourf(z);  
>>title('ezcountourf');
```

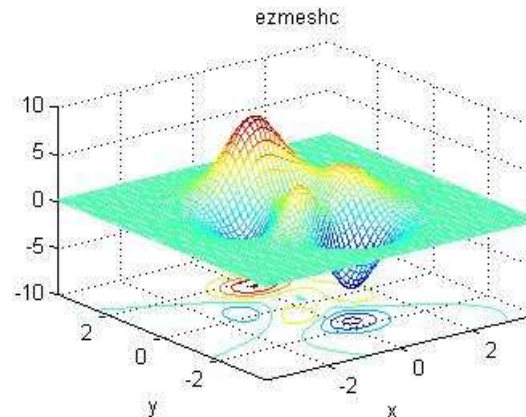


Grafici 3d

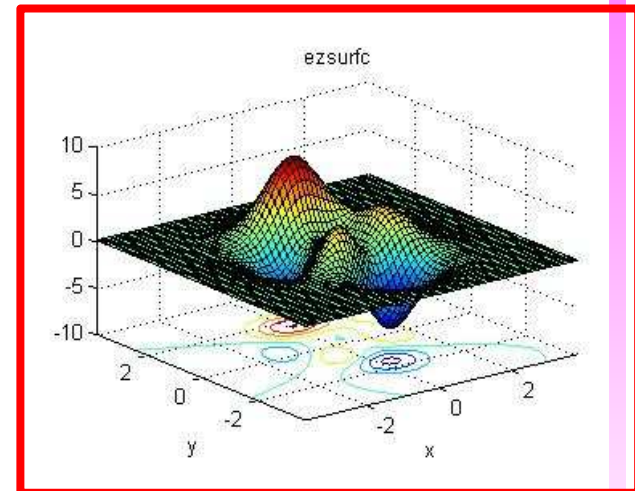


```
>>z1=sym('3*(1-x)^2*exp(-(x^2)-(y+1)^2)');  
>>z2=sym('-10*(x/5-x^3-y^5)*exp(-x^2-y^2)');  
>>z3=sym('-1/3*exp(-(x+1)^2-y^2)');  
>>z=z1+z2+z3;
```

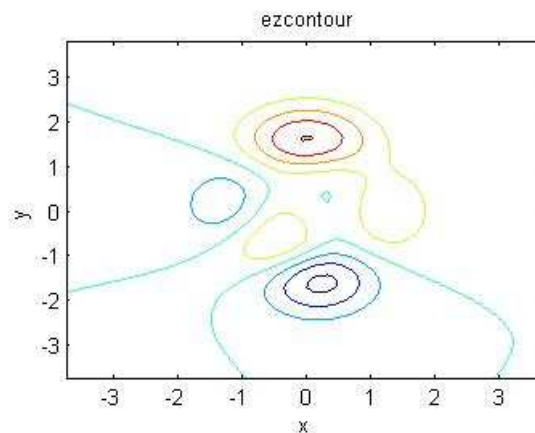
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>>subplot(2,2,1);  
>>ezmeshc(z);  
>>title('ezmeshc');
```



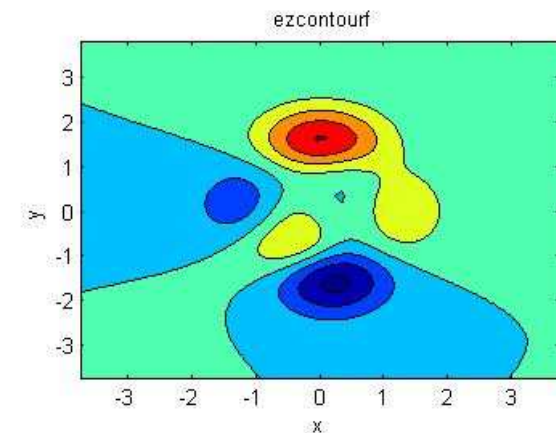
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>>subplot(2,2,2);  
>>ezsurf(z);  
>>title('ezsurf');
```



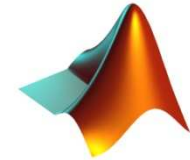
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>>subplot(2,2,3);  
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```
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>>title('ezcountourf');
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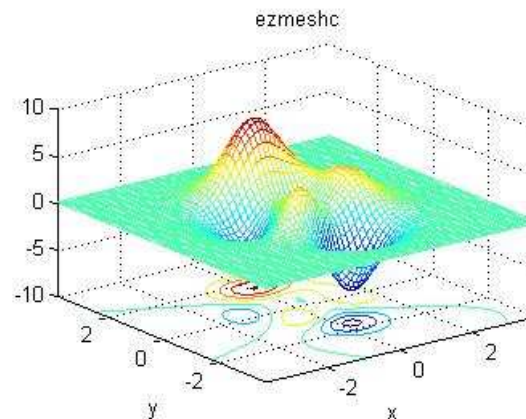


Grafici 3d

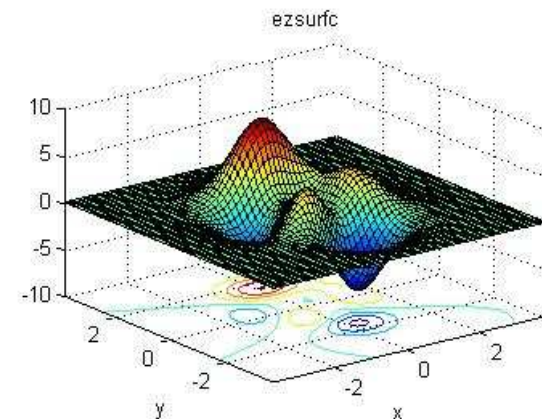


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>>z1=sym('3*(1-x)^2*exp(-(x^2)-(y+1)^2)');  
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>>z=z1+z2+z3;
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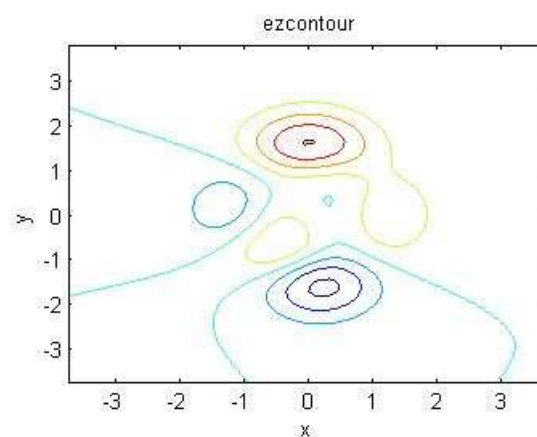
```
>>subplot(2,2,1);  
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>>title('ezmeshc');
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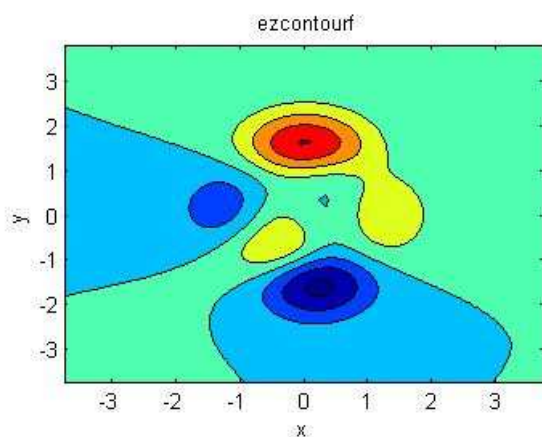
```
>>subplot(2,2,2);  
>>ezsurf(z);  
>>title('ezsurf');
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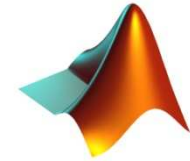
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>>ezcountourf(z);  
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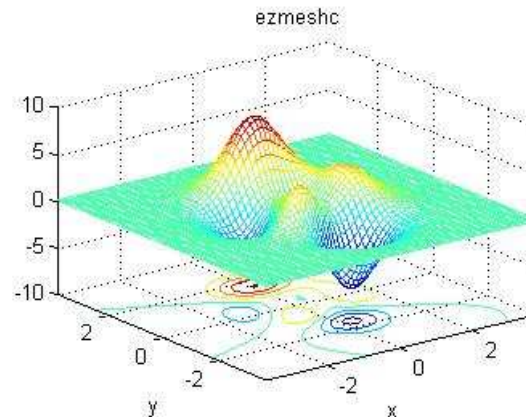


Grafici 3d

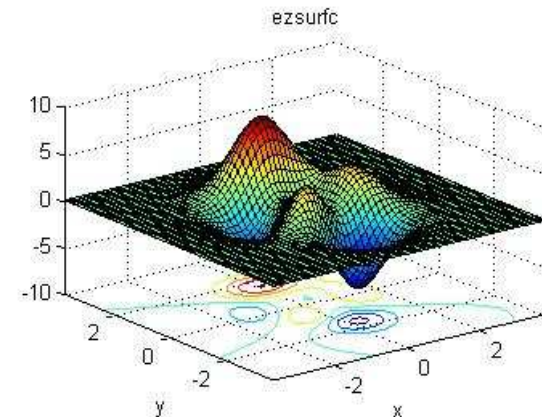


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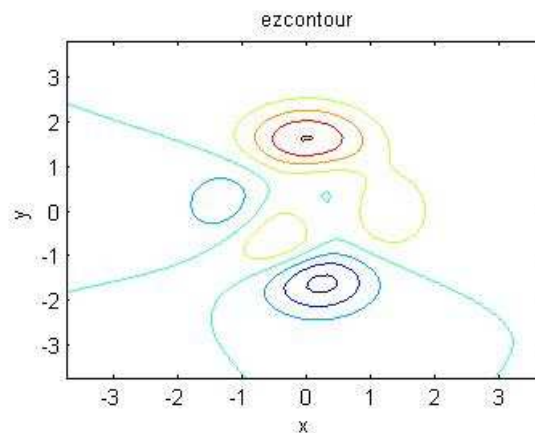
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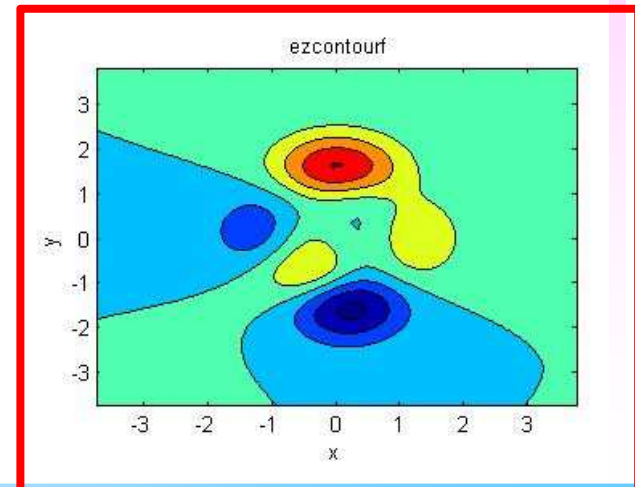
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```



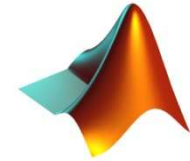
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>>ezcontour(z);  
>>title('ezcontour');
```



```
>>subplot(2,2,4);  
>>ezcountourf(z);  
>>title('ezcountourf');
```



Calcolatrice grafica interattiva



Il comando funtool fornisce una interfaccia grafica interattiva per manipolare simbolicamente delle funzioni e studiarne le principali caratteristiche

>> funtool

