AIM:
The aim of the project is the re-engineering, validation, and technology transfer of software for the prediction of femoral neck fracture in osteoporotic patients at the clinical site.

PROBLEM STATEMENT:
Femoral neck fractures are an increasing world-wide clinical and social problem because of their correlation to the population ageing. In particular, the hip fractures are one of the most important causes of morbidity and mortality in the elderly population. For these reasons, accurately predict the risk of femoral neck fracture is an important objective.

A threshold on the bone mineral density was defined and used in the daily clinical practice. However, the density distributions of at-risk patients and age-matched controls present a large overlap reducing the classification accuracy to 60-70%. To improve the accuracy in the prediction, new software for the evaluation of a global risk indicator was developed.

In the finite element simulation (approximate) solutions of the equilibrium of linear elasticity theory are calculated. In other words the equation

\[ -\text{div} \, S = 0 \]

is solved numerically. Here \( S \) denotes the stress and is given by the application of the elasticity tensor \( C \) on the strain tensor \( \varepsilon \):

\[ S = C \varepsilon = \lambda \varepsilon + 2\mu \varepsilon. \]

In the linearized theory applied here the strain tensor \( \varepsilon \) is the symmetrized gradient of the displacement field \( u \):

\[ \varepsilon = \frac{1}{2} \left( \nabla u + \nabla u^T \right). \]

The Lamé coefficients \( \lambda \) and \( \mu \) are not constant in the domain but are given by

\[ \lambda = \frac{E(x)}{(1 + \nu)(1 - 2\nu)} \quad \mu = \frac{E(x)}{2(1 + \nu)} \]

where the Young modulus \( E \) is assumed to be proportional to the bone density as measured by the DXA-scan. The poisson ration \( \nu \) is set constant to the value \( \frac{1}{3} \).

The external load can be either modelled via Dirichlet boundary conditions or by Neumann boundary conditions.

FINITE ELEMENT SIMULATION:
The weak formulation of the elastic system is:

\[ -\text{div} \, \varepsilon = 0 \]

for each \( \xi \in (H^1(\Omega))^3 \). If \( S^h \) is the space of piecwise linear, continuous functions the finite element problem reads:

\[ \int_{\Omega} \varepsilon(x)^T \xi(x) dx = 0 \]

for each \( \xi \in (S^h)^3 \).

A POSTERIORI ERROR ESTIMATORS:
For the grid refinement process we apply a posteriori error estimators that are due to Rannacher and Suttmeier. For a given error functional \( J \) the error is bounded by

\[ J(u - u^h) \leq \sum_{\text{all elements } s} \omega_s r_s \]

with the local residuals

\[ r_s = h \left| \varepsilon(x) \right|_{L^2(\Omega)} \frac{1}{2} \left[ n \cdot \left( \xi(x), \varepsilon(x)^T u^h \right) \right] \]

and weights

\[ \omega_s = \max \left\{ \| h^{-1} \|_{L^2(\Omega)} \| \cdot \|_{L^2(\Omega)} \right\} \]

where \( x \) is the solution of the corresponding dual problem and \( \omega_s \) is an approximation thereof.

SOLUTION OF THE LINEAR SYSTEM:
We use the preconditioned conjugate gradient method to solve the resulting linear system. As a preconditioner we apply the incomplete Cholesky factorization.

\[ \| M^{-1} \|_{\infty,\infty} = \max_{\text{all elements } s} \omega_s \]

The best choice for \( p \)-level of fill-in in terms of efficiency is \( p=3 \) Total time 0.086sec compared with 0.266sec without precondition (PENTIUM IV 1.5GHz).

OUTPUT AND CLASSIFICATION:
The quality of interest of the solution of the elastic system is the maximum principal strain in the neck region. This is evaluated by a linear Bayes classifier based on a retrospective population of 200 patients.

COSTUMI:
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- University of Bologna (DEIS and CIRAM), Italy; University of Duisburg, Germany
- Microidea, Italy