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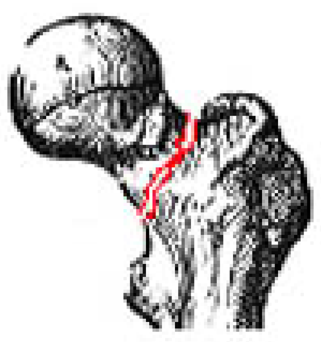
FRAFEM

Real-time software for the femoral neck fracture prediction

Aim:



The aim of the project is the re-engineering, validation, and technology transfer of software for the prediction of femoral neck fracture in osteoporotic patients at the clinical site.

Problem statement:

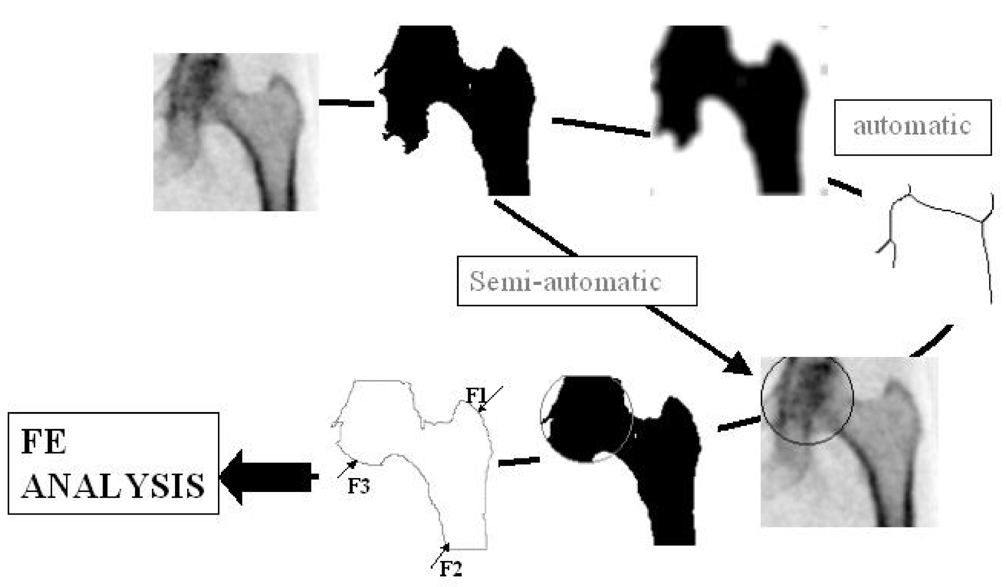
Femoral neck fractures are an increasing world-wide clinical and social problem because of their correlation to the population ageing. In particular, the hip fractures are one of the most important causes of morbidity and mortality in the elderly population. For these reasons, accurately predict the risk of femoral neck fracture is an important objective.

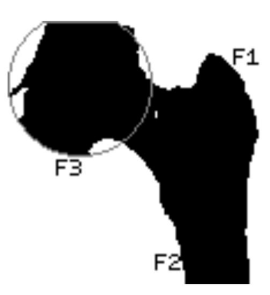
A threshold on the bone mineral density was defined and used in the daily clinical practice. However, the density distributions of at-risk patients and age-matched controls present a large overlap reducing the classification accuracy to 60-70%. To improve the accuracy in the prediction, new software for the evaluation of a global risk indicator was implemented and retrospectively validated showing an accuracy of about 80%.

These encouraging results were achieved by combining numerical analysis, mechanics of bone fracture, and biomechanics of fall with standard densitometry and statistical classification. Software was completely developed in a research context and was not adequate for a real clinical context since it required a high manual interaction and it was too time-consuming. Additionally, for the clinical context the application required higher robustness, and efficiency. For this reason, a re-engineering of the software was necessary to become a ”real-time” application.

Contour detection, extraction and load setting:

The proximal femur contour is extracted by a segmentation process. A number of contour points (ca. 200) is extracted which better preserves the geometry



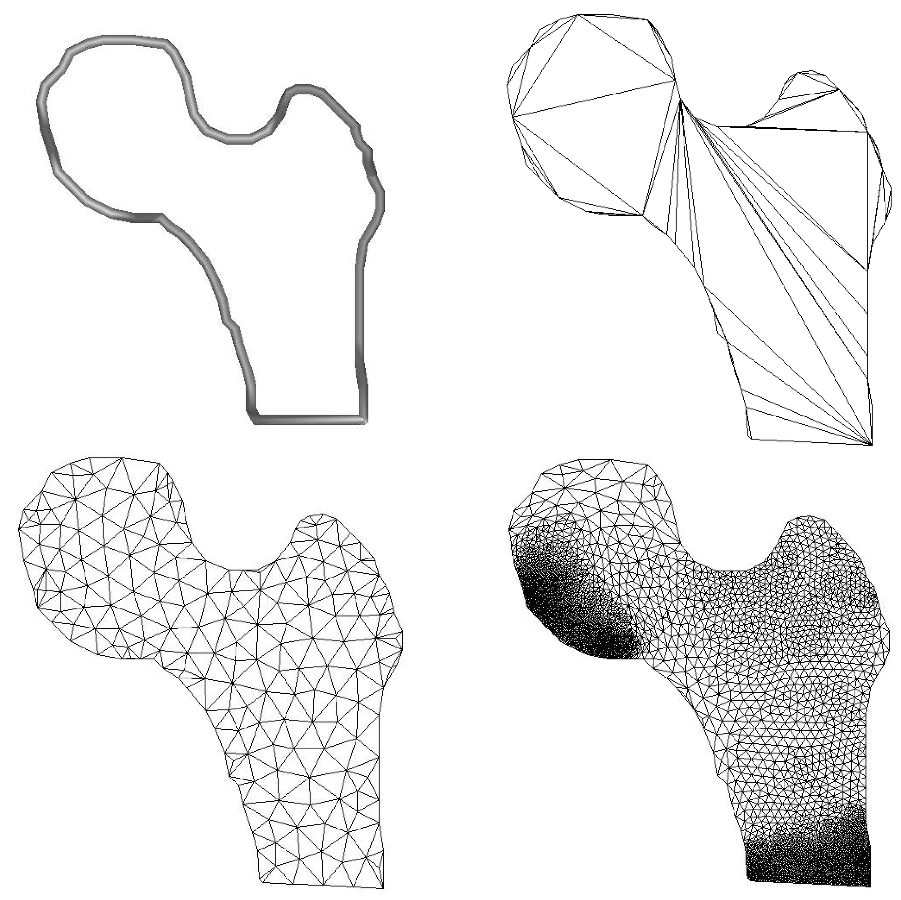


An automatic / semi-automatic detection of some anatomical landmarks in the femoral region for determining the directions of the loads/forces and the points of applications is applied.

Especially the direction of neck, diaphysis and the position of the head centre are estimated.

Mesh generation and refinement:

From the contour line a first grid is generated via a Delaunay algorithm. This grid is refined by inserting nodes within the contour and equilibrating triangle sizes and angles. This yields a coarse starting grid for the finite element simulation. In an iterative procedure the mesh is refined according to local a posteriori error estimators dependend on the results of the calculation.



The elastic system:

In the finite element simulation (approximate) solutions of the equilibrium of linear elasticity theory are calculated. In other words the equation

$$-\operatorname{div} S = 0$$

is solved numerically. Here S denotes the stress and is given by the application of the elasticity tensor C on the strain tensor ε:

$$S = C[\mathcal{E}] = \lambda tr(\mathcal{E}) Id + 2\mu\mathcal{E}.$$

In the linearized theory applied here the strain tensor ε is the symmetrized gradient of the displacement field u:

$$\mathcal{E} = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T).$$

The Lamé coefficients λ and μ are not constant in the domain but are given by

$$\lambda = \frac{E(x)\nu}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E(x)}{2(1+\nu)}$$

where the Young modulus E is assumed to be proportional to the bone density as measured by the DXA-scan. The poisson ration ν is set constant to the value $\frac{1}{3}$.

The external load can be either modelled via Dirichlet boundary conditions or by Neumann boundary conditions.

Finite element simulation:

The weak formulation of the elastic system is:

$$-\int_\Omega \operatorname{div}\Big(\mathcal{C}(x)[\mathcal{E}(\mathbf{u})]\Big)\cdot \boldsymbol{\xi}\,dx = \int_\Omega \mathcal{E}(\boldsymbol{\xi}) : \mathcal{C}(x)[\mathcal{E}(\mathbf{u})]\,dx = 0.$$

for each $\boldsymbol{\xi} \in (H^{1,2})^2(\Omega)$. If S^h is the space of piecewise linear, continuous functions the finite element problem reads:

Find $\mathbf{u}^h \in (S^h)^d$ *,such that*

$$\int_\Omega \mathcal{E}(\boldsymbol{\xi}^h) : \mathcal{C}(x)[\mathcal{E}(\mathbf{u}^h)]\,dx = 0$$

for each $\boldsymbol{\xi}^h \in (S^h)^d$.

A posteriori error estimators:

For the grid refinenment process we apply a posteriori error estimators that are due to Rannacher and Suttmeier. For a given error functional J the error is bounded by

$$\left|J(\mathbf{u}-\mathbf{u}^h)\right| \le \sum_{\text{all elements s}} \omega_s\rho_s$$

with the local residuals

$$\rho_s = h\left\|\operatorname{div}\mathcal{C}(x)[\mathcal{E}(\mathbf{u}^h)]\right\|_{L^2(s)} + \frac{1}{2}\sqrt{h}\left\|n\cdot\Big[\mathcal{C}(x)\left[\mathcal{E}(\mathbf{u}^h)\right]\Big]\right\|_{L^2(\partial s)}$$

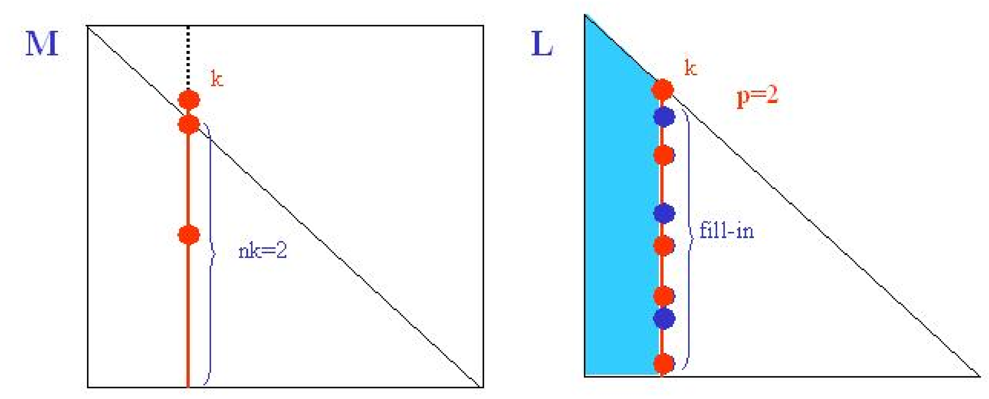
and weights

$$\omega_s = \max\left\{h^{-1}\left\|z-z^h\right\|_{L^2(s)}, h^{-\frac{1}{2}}\left\|z-z^h\right\|_{L^2(\partial s)}\right\}$$

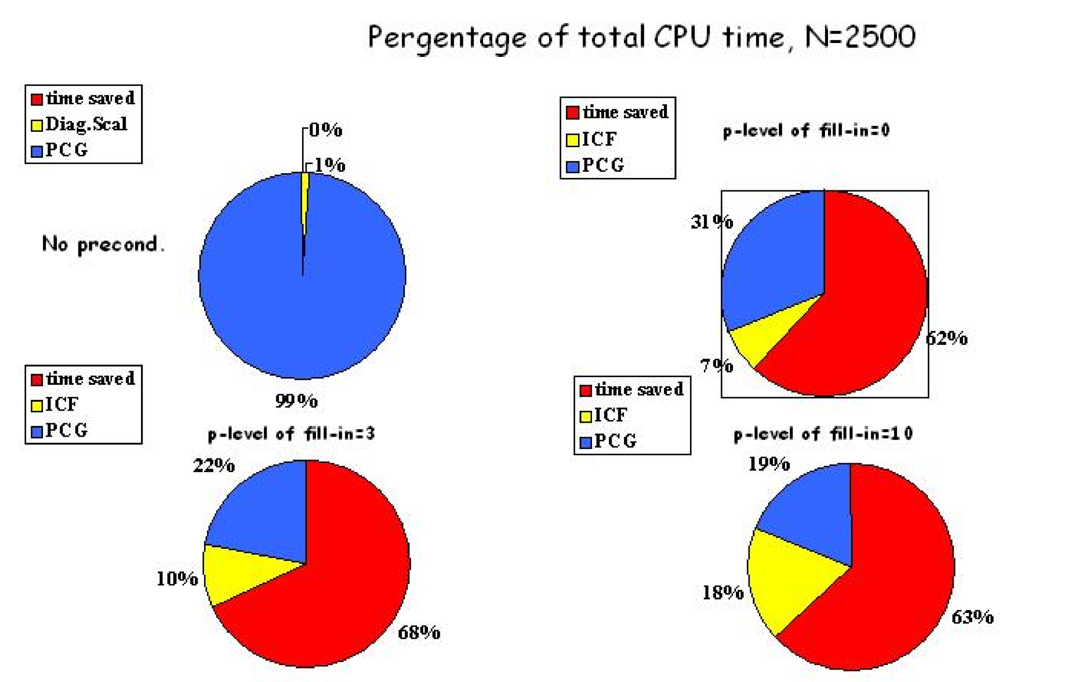
where z is the solution of the corresponding dual problem and z^h an approximation thereof.

Solution of the linear system:

We use the preconditioned conjugate gradient method to solve the resulting linear system. As a preconditioner we apply the incomplete Cholesky factorization.

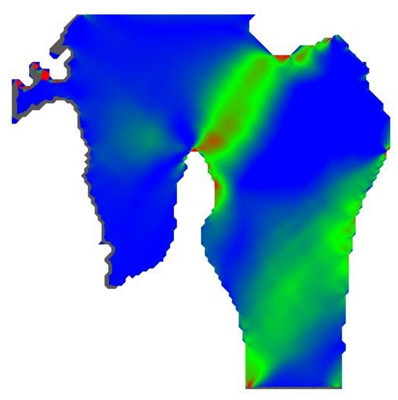


Here the incomplete factor L of the positive definit and symmertic matrix M is built column by column. Memory is the only criterion for dropping elements: only the nk+p largest module elements are retained. The numerical results below show the pergentage of total CPU time for a system with N=2500 unknowns.



The best choice for p-level of fill-in in terms of efficiency is p=3 Total time 0.086sec compared with 0.266sec without precond (PENTIUM IV 1.5GHz).

Output and classification:



The quantity of interest of the solution of the elastic system is the maximum principal strain in the neck region. This is evaluated by a linear Bayes classifier based on a retrospective population of 200 patients.

Consortium:

- CINECA, Super-computing Centre, Italy
- Istituti Ortopedici Rizzoli (Laboratorio Tecnologia Medica and Servizio Medicina Interna), Italy
- University of Bologna (DEIS and CIRAM), Italy;
- University of Duisburg, Germany
- MicroIdea, Italy