$$f(x) = \left(\frac{x^2}{2} - x\right)\left(\ln x - \frac{5}{2}\right) + \frac{x}{2}$$

$$\mathbb{D}(f) = \{x > 0\}$$

$$\lim_{x \to 0+} f(x) = 0 \quad \text{siccome} \quad \lim_{x\to 0+} x \ln x = 0$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$f'(x) = (x-1)\left(\ln x - 5/2\right) + \left(\frac{x^2}{2} - x\right)\frac{1}{x} + \frac{1}{2}$$

$$= (x-1)\left(\ln x - \frac{5}{2}\right) + \frac{x}{2} - \frac{1}{2}$$

$$= (x-1)\left(\ln x - \frac{5}{2} + \frac{1}{2}\right) = (x-1)(\ln x - 2)$$

$x - 1 > 0 \longrightarrow x > 1$

$\ln x - 2 > 0 \longrightarrow x > e^2$

	0		1		e^2	
$x-1$		$-$		$+$		$+$
$\ln x - 2$		$-$		$-$		$+$
f'		$+$		$-$		$+$
f		$\nearrow$		$\searrow$		$\nearrow$

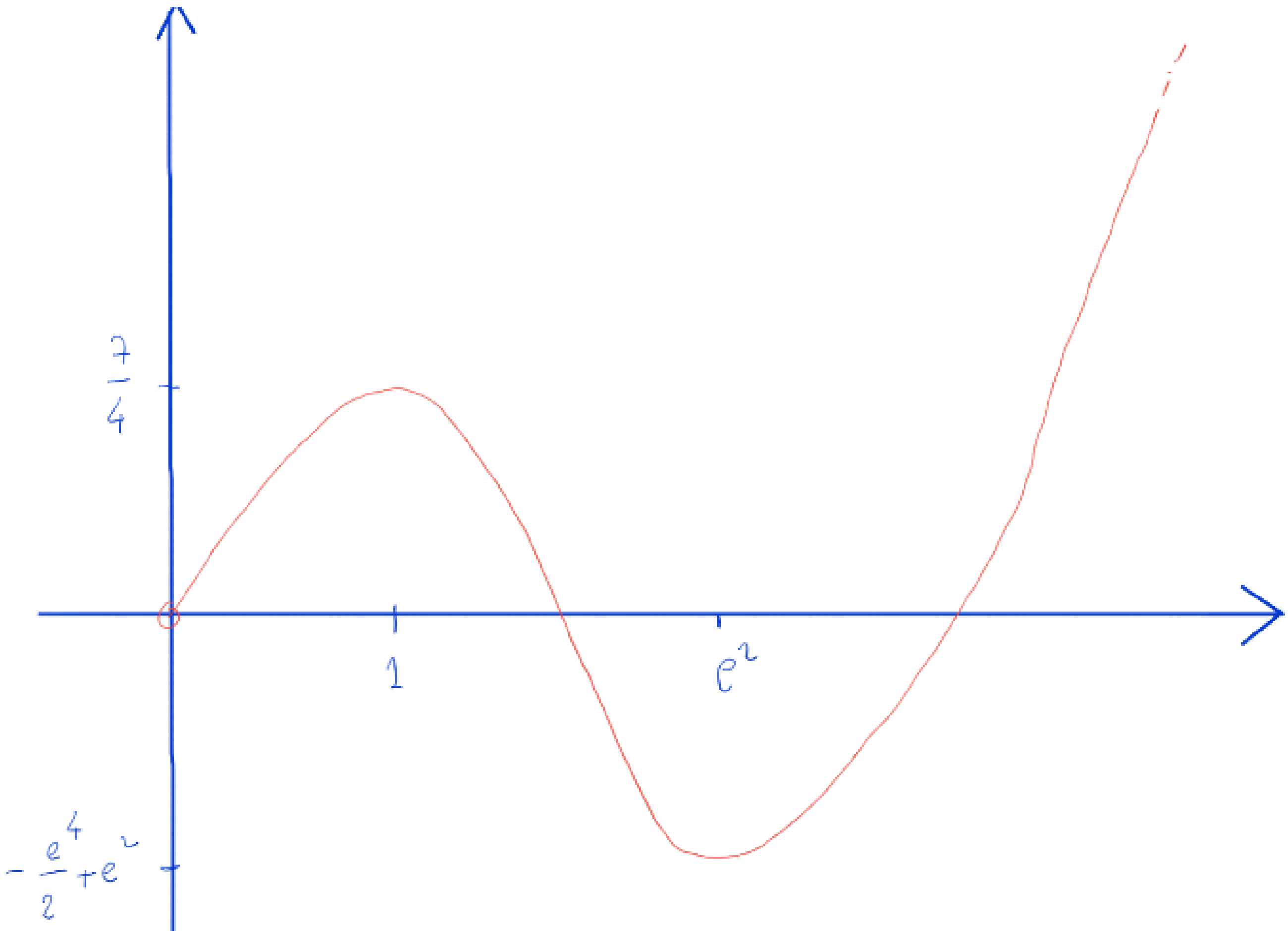
$$f(1) = \left(\frac{1}{2} - 1\right)\left(-\frac{5}{2}\right) + \frac{1}{2} = \frac{7}{4}$$

$$f(e^2) = \left(\frac{e^4}{2} - e^2\right)\left(2 - \frac{5}{2}\right) + \frac{e^2}{2} = -\frac{e^4}{2} + e^2 < 0$$

$$-\frac{e^4}{2} + e^2 < 0 \iff 1 - \frac{e^2}{2} < 0 \iff e > \sqrt{2}$$

OK dato che

$2 < e < 3$, $\sqrt{2} < 2$



$f(x)=\lambda$ ha 2 soluzioni

$\Updownarrow$

$$-\frac{e^4}{2}+e^2 < \lambda \le 0 \quad \lor \quad \lambda = \frac{7}{4}$$

2 $$\lim_{x\to 0} \frac{\sin(\ln(x+1)) - \ln(x+1) + \frac{x^3}{6}}{\sin^4 x}$$

$$\sin^4 x = x^4 + o(x^4)$$

$$\sin(\ln(1+x)) = \ln(1+x) - \frac{1}{6}(\ln(1+x))^3 + \underbrace{o\left((\ln(1+x))^4\right)}_{= o(x^4)}$$

$$\sin\left(\ln(1+x)\right) - \ln(1+x) = -\frac{1}{6}\left(\ln(1+x)\right)^3 + o(x^4)$$

$$= -\frac{1}{6}\left(x - \frac{x^2}{2} + o(x^2)\right)^3 + o(x^4) =$$

$$= -\frac{1}{6}\left(x^3 - \frac{3}{2}x^4 + o(x^4)\right) = -\frac{x^3}{6} + \frac{x^4}{4} + o(x^4)$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\frac{x^4}{4} + o(x^4)}{x^4 + o(x^4)} = \frac{1}{4}$$

③ $$\int_0^1 4x^3 \arctan x \, dx$$

$$\int 4x^3 \arctan x \, dx = x^4 \arctan x - \int \frac{x^4}{x^2+1} dx$$

$$\left(\frac{x^4}{x^2+1} = \frac{x^4 - 1 + 1}{x^2+1} = x^2 + 1 + \frac{1}{x^2+1}\right)$$

$$= x^4 \arctan x - \frac{x^3}{3} + x - \arctan x$$

$$= (x^4 - 1)\arctan x - \frac{x^3}{3} + x$$

$$\int_0^1 4x^3 \arctan x \, dx = -\frac{1}{3} + 1 = \frac{2}{3}$$

4

$$f(x,y) = x^3 + 3xy^2 - 15x + 12y$$

$$\begin{cases} f_x = 3x^2 + 3y^2 - 15 = 0 \\ f_y = 6xy + 12 = 0 \end{cases} \quad \begin{cases} x^2+y^2 = 5 \\ xy = -2 \end{cases}$$

Si risolve usualmente per sostituzione oppure:

$$5 = x^2+y^2 = (x+y)^2 - 2xy = (x+y)^2 + 4$$

$$\begin{cases} (x+y)^2 = 1 \\ xy = -2 \end{cases} \quad \begin{cases} x+y = \pm 1 \\ xy = -2 \end{cases}$$

$$t^2 + t - 2 = 0 \qquad \Delta = 1+8 = 9$$

$$t_{1,2} = \frac{-1 \pm 3}{2} = \begin{matrix} -2 \\ 1 \end{matrix} \qquad \underset{A_1}{\begin{cases} x = 1 \\ y = -2 \end{cases}} \quad \underset{A_2}{\begin{cases} x = -2 \\ y = 1 \end{cases}}$$

$$t^2 - t - 2 = 0$$

$$t_{1,2} = \frac{1 \pm 3}{2} \begin{matrix} -1 \\ 2 \end{matrix} \qquad \underset{A_3}{\begin{cases} x = -1 \\ y = 2 \end{cases}} \quad \underset{A_4}{\begin{cases} x = 2 \\ y = -1 \end{cases}}$$

$$H_f(x,y) = \begin{pmatrix} 6x & 6y \\ 6y & 6x \end{pmatrix}$$

$$H_f(A_1) = \begin{pmatrix} 6 & -12 \\ -12 & 6 \end{pmatrix} \qquad \det H_f(A_1) = 36 - 144 < 0$$

SELLA

$$H_f(A_2) = \begin{pmatrix} -12 & 6 \\ 6 & -12 \end{pmatrix} \quad \text{det } H_f(A_2) = 144 - 36 > 0$$

$-12 < 0$

p. DI MAX

$$H_f(A_3) = \begin{pmatrix} -6 & 12 \\ 12 & -6 \end{pmatrix} \quad \text{p. DI SELLA}$$

$$H_f(A_4) = \begin{pmatrix} 12 & -6 \\ -6 & 12 \end{pmatrix} \quad \text{p. DI MIN.}$$

$12 > 0$

piano tangente in $(-2,-1)$

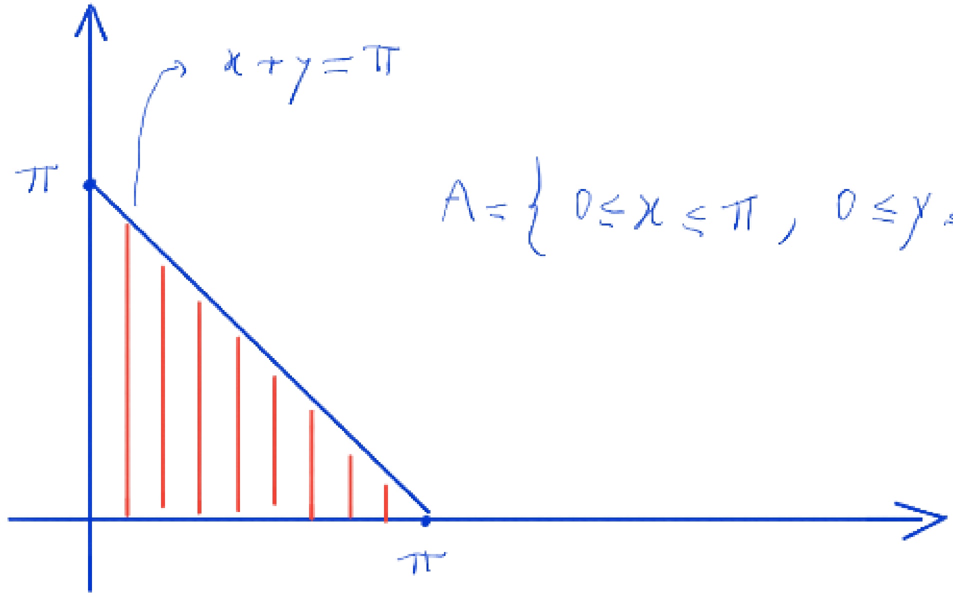
$$f(-2,-1) = -8 - 6 + 30 - 12 = 4$$

$$z = \left< \nabla f(-2,-1), \begin{pmatrix} x+2 \\ y+1 \end{pmatrix} \right> + f(-2,-1)$$

$$= \left< \begin{pmatrix} 0 \\ 24 \end{pmatrix}, \begin{pmatrix} x+2 \\ y+1 \end{pmatrix} \right> + 4$$

$$= 24y + 28$$

5



$$A = \{ 0 \leq x \leq \pi,\ 0 \leq y \leq \pi - x \}$$

$$\iint_A \cos x \cdot \cos y \, dx\, dy =$$

$$= \int_0^{\pi} \int_0^{\pi - x} \cos x \cdot \cos y \, dy\, dx = \int_0^{\pi} \cos x \cdot \left[\int_0^{\pi - x} \cos y \, dy \right] dx$$

$$= \int_0^{\pi} \cos x \left[\sin y \right]_0^{\pi - x} dx =$$

$$= \int_0^{\pi} \cos x \cdot \underbrace{\sin(\pi - x)}_{\left(\sin \pi \cdot \cos x - \cos \pi \cdot \sin x = \sin x \right)} dx =$$

$$= \int_0^{\pi} \cos x \cdot \sin x \, dx = (*)$$

I METODO:

$$\cos x \cdot \sin x = \frac{1}{2} \sin 2x$$

$$(*) = \frac{1}{2}\int_0^{\pi} \sin 2x \, dx = \left[-\frac{1}{4}\cos 2x\right]_0^{\pi} = 0$$

II METODO:

Integrando per parti:

$$\int \cos x \cdot \sin x \, dx = \sin^2 x - \int \sin x \cdot \cos x \, dx$$

$$2\int \cos x \cdot \sin x \, dx = \sin^2 x$$

$$\Rightarrow \int \cos x \cdot \sin x \, dx = \frac{1}{2}\sin^2 x$$

$$\int_0^{\pi} \cos x \cdot \sin x \, dx = \frac{1}{2}\left[\sin^2 x\right]_0^{\pi} = 0$$

III METODO:

$$\int \cos x \cdot \sin x \, dx \qquad t = \sin x$$

$$= \int t \, dt = \frac{t^2}{2} = \frac{\sin^2 x}{2} \qquad dt = \cos x \, dx$$

Ultima modifica: 11:42