ERRATA CORRIGE

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PDE and Martingale Methods in Option Pricing

Andrea Pascucci
http://www.dm.unibo.it/∼pascucci/

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Chapter 2

p.53, +6: ...“covers” the whole $\mathbb{R}_{>0}$ as $N \to \infty$.

p.68, +12: ...measure P.

Chapter 3

p.131, -3: if $T \notin \varsigma$ then..

Chapter 4

p.149, Exercise 4.16: Under the hypotheses of Theorem 4.11, prove that for $\mathcal{G} \subseteq \mathcal{F}_a$ we have in general

$$E\left[\int_a^b u_t dW_t \mid \mathcal{G}\right] \neq \int_a^b E\left[u_t \mid \mathcal{G}\right] dW_t.$$

p.154, -1:

$$= E\left[\int_s^t u_\tau^2 d\tau - \langle X \rangle_t \mid \mathcal{F}_s\right] + X_s^2 = M_s^2 - \langle X \rangle_s.$$

Chapter 6

p.206, -8: **Proof.** First of all, by Remark 6.8 ....

p.212, -2: Indeed we have $u(t_0, x) = 0$ and

Chapter 7

p.222, 10:

$$d\widetilde{V}_t^{(\alpha,\beta)} = \sigma \widetilde{S}_t \alpha_t dW_t$$

Chapter 8

p.262: estimates (8.13)-(8.14) of Theorem 8.10 hold true under the additional regularity Hypothesis 8.13. In general, under Hypotheses 8.1 and 8.3, we have

$$|\partial_{x_i}\Gamma(t,x;s,y)| \leq \frac{C}{\sqrt{t-s}}\Gamma_{\lambda+\varepsilon}(t-s,x-y),$$
$$\left|\partial_{x_i x_j}\Gamma(t,x;s,y)\right| + |\partial_t\Gamma(t,x;s,y)| \leq \frac{C}{t-s}\Gamma_{\lambda+\varepsilon}(t-s,x-y).$$

Chapter 9

p.306, -11: Formula (9.55) is related to the classical *method of characteristics* which can be used to solve the initial value problem for general first order (containing only first order partial derivatives) PDEs

p.314, -2:

$$\begin{cases} \Phi'(t) = B(t)\Phi(t), \\ \Phi(0) = I_N, \end{cases}$$

p.316, formula (9.75):

$$L = \frac{1}{2}\sum_{i,j=1}^{N} c_{ij}(t)\partial_{x_i x_j} + \sum_{i=1}^{N} b_i(t)\partial_{x_i} + \sum_{i=1}^{N} B_{ij}(t)x_j\partial_{x_i} + \partial_t$$

p.316, 7: The case of constant coefficients...

Chapter 10

p.341, 4:

$$\varrho^{ii} = \sum_{j=1}^{d} \left(A^{ij}\right)^2 = 1.$$

Then, by Corollary 5.35,

$$W_t^i = \sum_{j=1}^{d} A^{ij}\bar{W}_t^j, \qquad 1 = 1, \dots, d,$$

is a standard 1-dimensional Brownian motion...

p.371, 9: Therefore, assuming the dynamics (10.92), we get

$$d\gamma_t^i = \frac{p_0^i}{p_0^{i-1}}\delta_i L_t^i \sigma_t^i dW_t^{i,i} = \gamma_t^i \sigma_t^i \frac{\delta_i L_t^i}{1+\delta_i L_t^i} dW_t^{i,i}.$$

p.371, formula (10.96):

$$\mu_t^i = -\sigma_t^i \sum_{k=i+1}^{N} \varrho^{ik}\sigma_t^k \frac{\delta_k L_t^k}{1+\delta_k L_t^k}, \qquad i < N,$$

p.371, -2:

$$dL_t^i = -L_t^i \sum_{k=i+1}^{N} \varrho^{ik}\sigma_t^i\sigma_t^k \frac{\delta_k L_t^k}{1+\delta_k L_t^k} dt + \sigma_t^i L_t^i dW_t^{N,i}, \qquad i = 1, \dots, N-1,$$

p.375, 6:

$$= E^Q\left[e^{-\int_0^T r_s ds}(S_T - K)\mathbb{1}_{\{S_T \geq K\}}\right] = I^1 - I^2$$

p.375, 10: by the change of numeraire...

p.376, 14:

$$\frac{dQ^T}{dQ}|_{\mathcal{F}_t^W} = \frac{p(t,T)B_0}{B_t p(0,T)}.$$

p.380, 7: the SDE (10.111) has a unique strong solution for any $\beta \in [0,1]$ and not only for $\beta \geq \frac{1}{2}$. A proof of the fact that

$$u(t,s) = E\left[(S_T^{t,s} - K)^+\right]$$

solves the Cauchy problem (10.112) can be found in

Janson, S. and Tysk, J. Feynman-Kac formulas for Black-Scholes-type operators. *Bull. London Math. Soc.* 38 (2006), no. 2, 269282

p.382, 2:

$$F_t = \frac{e^{r(T-t)} S_t + K}{2}.$$

p.383, formula (10.120):

$$\begin{cases} \partial_\tau Q - \frac{1}{2}\partial_{xx} Q = \varepsilon x \nu_1 \partial_{xx} Q + \frac{\varepsilon^2 x^2 (\nu_1^2 + \nu_2)}{2} \partial_{xx} Q + \mathrm{O}(\varepsilon^3), & \tau > 0,\ x > -\frac{K}{\varepsilon}, \\ Q(0,x) = x^+, \qquad x > -\frac{K}{\varepsilon}. \end{cases}$$

p.383, -4:

$$\begin{cases} \partial_t \widetilde{Q} - \frac{1}{2}\partial_{xx} \widetilde{Q} = \varepsilon x \nu_1 \partial_{xx} \widetilde{Q} + \frac{\varepsilon^2 x^2 (\nu_1^2 + \nu_2)}{2} \partial_{xx} \widetilde{Q} + \mathrm{O}(\varepsilon^3), & t > 0,\ x > -\frac{K}{\varepsilon}, \\ \widetilde{Q}(0,x) = x^+, \qquad x > -\frac{K}{\varepsilon}, \end{cases}$$

p.384, formula (10.127):

$$\partial_t G(t,x) = \frac{1}{2}\Gamma(t,x), \quad \partial_x G(t,x) = \int_{-\infty}^x \Gamma(t,y)dy,$$

Chapter 13

p.434, formula (13.3):

$$P(N_t = n) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}, \qquad n \in \mathbb{N} \cup \{0\},$$

p.445, -8:

$$= \sum_{n \geq 1} P(N_t = n) \sum_{k=1}^{n} E\left[\delta_{Z_k}(H)\right]$$

p.447, formula (13.26):

$$= \lambda t E\left[f(Z_1)\right] = E\left[N_t f(Z_1)\right].$$

p.448, +2:

$$E\left[\int_s^t \int_{\mathbb{R}^d} f(x) J(d\varsigma, dx) \mid \mathcal{F}_s\right] = E\left[\sum_{N_s < n \leq N_t} f(Z_n)\right]$$

p.448, in the main formula $\sum\limits_{n \geq 1}$ should read $\sum\limits_{n \geq 0}$

p.451, formula (13.38):

$$t \mapsto J_t(H,f) := \int_0^t \int_H f(x) J(ds,dx) = \sum_{0 < s \leq t} f(\Delta X_s) \mathbb{1}_H(\Delta X_s)$$

p.453, delete the second line of formula (13.51): thus formula (13.51) reads

$$\widetilde{X}_t^{\varepsilon,R} = \int_0^t \int_{\varepsilon \leq |x| < R} x\left(J(ds,dx) - \nu(dx)ds\right)$$

p.454, formula (13.52):

$$\mu_R = \mu_S + \int_{S \leq |x| < R} x \nu(dx).$$

p.454, 16:

$$\mu_R t + \int_0^t \int_{S \leq |x| < R} x \widetilde{J}(ds,dx) = \mu_S t + \int_{S \leq |x| < R} x \nu(dx),$$

p.454, formula (13.53):

$$\int_{|x|<1} |x| \nu(dx) < \infty,$$

p.455, formula (13.54):

$$\mu_0 = \mu_1 - \int_{|x|<1} x\nu(dx) < \infty,$$

p.455, formula (13.57):

$$\mu_\infty := \lim_{R\to+\infty} \mu_R = \mu_S + \int_{|x|\geq S} x\nu(dx).$$

p.455, delete the second line of formula (13.58): formula (13.58) reads

$$X_t = \mu_\infty t + B_t + \int_0^t \int_{\mathbb{R}^d} x\widetilde{J}(ds, dx)$$

p.456, +5: If Z_1 is integrable then conditions (13.53) and (13.56) are satisfied...

p.456, +9:

$$\mu_1 = \mu + \lambda \int_{|x|<1} x\eta(dx);$$

p.456, +10: the ∞-triplet of X is $(\mu + \lambda E[Z_1], \mathcal{C}, \lambda\eta)$ (see also (13.73)).

p.458, +3: **Proof.** By the Lévy-Itô decomposition X_t is equal a.s. to the limit of the sum...

p.458, formula (13.63):

$$\mu_R = \mu_1 + \int_{1\leq|x|<R} x\nu(dx).$$

p.460, formula (13.67):

$$\mu_0 = \mu_1 - \int_{|x|<1} x\nu(dx).$$

p.460, formula (13.70):

$$c_2(X_t) = t\left(\sigma^2 + \int_{\mathbb{R}} x^2\nu(dx)\right),$$

p.461, formula (13.71):

$$c_n(X_t) = t\int_{\mathbb{R}} x^n\nu(dx), \qquad n \geq 3.$$

p.465, +3:

$$\Phi_{X_t}(x) = e^{-\lambda t}\sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!\sqrt{2\pi(\sigma^2 t + n\delta^2)}} e^{-\frac{(x-\mu t-nm)^2}{2(\sigma^2 t+n\delta^2)}}.$$

p.481, formula (13.91):

$$E^Q[S_T] = E^Q\left[S_0 e^{X_T}\right] < \infty$$

Chapter 14

p.499, +4: Thus in this case (i.e. when $T_1 \in \varsigma_n$ for any n) the Riemann-Stieltjes integral is well-defined...

p.524, +4: then we can let R go to zero in (14.45)

p.530, +16: Regularity of elliptic ($\sigma > 0$)...

Chapter 15

p.571, 5: Setting $\nu_0 = u_0$ and $\nu_\infty = u$, the first two cumulants are given by

p.572, -2: Here $S_0 = 130$, $K = 100$ and $T = 5$: the reference value, computed with $N = 1000$, is 4.424162989.

Chapter 16

p.593, +13:

$$\partial_\sigma S_T = (W_T - 2\sigma T)S_T, \qquad D_s S_T = \sigma S_T.$$

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