Modulation of periodic waves in systems of balance laws

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Periodic patterns and traveling waves arise quite generally in optics, biology, chemistry, and many other applications. A great success story over the past couple decades for the dynamical systems approach to PDE has been the rigorous treatment of modulation of periodic patterns in reaction diffusion systems. However, the techniques used were designed for modulations with a single degree of freedom. For systems possessing one or more conservation laws, hence two or more degrees of freedom- in particular, the Kuramoto-Sivashinsky, Saint Venant, and other equations governing thin film flow- these methods do not apply.

Here, we present a general approach applying to general systems of balance laws including both the reaction diffusion and conservation law case, rigorously verifying an associated "Whitham system" formally governing slow modulations under suitable numerically verifiable stability assumptions on the spectra of the linearized operator about the background pattern. This verifies/explains a number of numerically observed phenomena in thin film flow, including 'viscoelastic behavior" in cellular Kuramoto-Sivashinsky behavior, and the "homoclinic paradox" in inclined thin-film flow, the latter concerning the puzzling phenomenon that asymptotic behavior appears to consist of solitary waves, despite that solitary waves are readily seen to be exponentially unstable.

We conclude by discussing verification of the spectral assumptions in weakly and strongly unstable regimes, by a combination of asymptotic and numerical methods, and extensions to the case of *defect patterns* corresponding roughly to shock waves of the associated Whitham system.

Tentative schedule

Lecture I: *Turing patterns, Bloch decomposition, and stability*
Lecture II: *Basic nonlinear stability argument, localized data*
Lecture III: *Nonlocalized data/verification of the Whitham equation*
Lecture IV: *Systems of balance laws and the Whitham modulation system*
Lecture V: *Spectral stability and applications to thin film flow*
Lecture VI: *Extensions: stability of source defects*