Vacuum Dynamics for Compressible Fluids

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Abstract:

The evolution of vacuum states is important not only in physical problems such as motions of gaseous stars, but also in the theory of well posedness of solutions to the multi-dimensional compressible Navier-Stokes systems (or Euler systems). Due to the degeneracies and singularities in the presence of vacuum, there are many rich phenomena and challenging problems to be investigated. Many fundamental questions remain open, such as the validity of the Navier-Stokes models in the presence of vacuum; can vacuum state arise in finite time from non-vacuum data? the propagation of the so called physical vacuum; and qualitative behaviors of viscous (or ideal) fluids in the presence of vacuum, etc.. The goal of this short course is to give a brief introduction to the recent progress in the studies along these directions. We will try to illustrate some basic difficulties and new ideas involved in the recent main progress. As time allows, we will cover the following topics:

(1) Survey of the theory of small-amplitude non-vacuum smooth solutions and the existence theory of P.L. Lion's weak solutions to multi-dimensional compressible isentropic Navier-Stokes systems.
(2) Finite -time blow-up of classical solutions to the full multi-dimensional compressible Navier-Stokes system for a class of data with vacuum.
(3) Global in time existence of smooth solutions to the multi-dimensional isentropic compressible Navier-Stokes system with possible large oscillations and vacuum, and the uniqueness and regularity of P.L. Lions weak solutions.
(4) On the expanding of gases into vacuum with/or without self-gravitation, and nonlinear asymptotic stability of Lane-Emden solutions for the viscous gaseous star motions.
(5) Studies of the compressible Navier-Stokes systems with density-dependent viscosities.
(6) Open problems.