RiemannHilbert boundary value problems and applications to random matrices, random processes, integrable systems.

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The course is aimed at presenting a host of methods that go under the umbrella name of "Riemann–Hilbert methods" with a wide range of applications.

These are a particular type of boundary value problems in the complex plane for matrix valued functions. They find applications to several areas of mathematical physics like

- the inverse scattering method for integrating nonlinear integrable wave equations;
- orthogonal and multiple orthogonal polynomials;
- computations of Fredholm determinants of certain integral operators;
- random matrices;
- asymptotic analysis of orthogonal polynomials;
- determinantal random processes.
- Painlevé equations.

The course aims at giving first a short overview of these applications and then focusing on specific applications to random matrices and orthogonal polynomials, entering in a certain level of detail of the so-called Deift–Zhou method for analyzing asymptotic behavior of Riemann Hilbert problem depending on parameters.

A rough breakdown of the course is as follows

1. Introduction and overview of applications;
2. Random Matrices: a primer. Connection with Orthogonal polynomials. Determinantal Random Point Fields and Processes.
3. Riemann Hilbert problems: generalities.
4. Asymtpotics of Riemann Hilbert problems and orthogonal polynomials.
5. The Deift-Zhou method of asymptotic analysis.