Pattern formation in reaction-diffusion systems - an explicit approach

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In pattern formation the complex dynamics of nonlinear spatial processes is studied. Reaction-diffusion systems appear as relevant models for such processes and are at the core of the mathematical analysis of pattern formation. In these lectures we will present an introduction to the basic mathematical 'tools' by which an understanding of complex pattern dynamics can be built. For simplicity, we will focus on 2-component reaction-diffusion systems in one spatial dimension. We will first consider the emergence of patterns `near onset' by the Ginzburg-Laundau approximation scheme. The main topic of the course will be the analysis of `far from equilibrium patterns'. Such patterns can be studied explicitly -- i.e. in full analytical detail -- by assuming that the reaction-diffusion system is singularly perturbed, which means that one of the components diffuses much faster than the other (measured by a `sufficiently small' parameter ε). The backbone of far from equilibrium dynamics is given by localized patterns. These patterns correspond to homoclinic of heteroclinic orbits in the 4 – dimensional (spatial) ODE reduction associated to the reaction-diffusion system. Geometric singular perturbation theory (GPST) is crucial to understanding and rigorously establishing the existence of these orbits. Of course, the existence analysis must necessarily be followed by a spectral stability analysis (in the context of the full PDE). Again the combination of the geometrical point of view with the singularly perturbed nature of the problem enables one to explicitly determine the spectrum associated to the pattern -- by a combination of GPST and the Evans function approach. The course will be concluded by an introduction to further methods and ideas by which more complex patterns in more complex systems may be understood.