Free surface singularities

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Interfacial flows are of particular interest both mathematically and from a physical perspective, because they exhibit and unusually rich variety of singular phenomena. Examples are drop formation, coalescence, contact line motion, and free surface tips and cusps.

I begin with a pedagogical introduction to self-similar phenomena and scaling. Since singularities are associated with a vanishing length scale, they are described by similarity solutions: upon rescaling profiles collapse onto a universal similarity profile. This will be illustrated using three different examples: optics (wave fronts), thin film dynamics, and the thin jet equations. I will discuss the role of dimensionless control parameters, and then look at the stability of solutions.

In a second part, I would like to discuss more recent developments: singular phenomena in higher dimensions and randomness.
Most known solutions describe a one-dimensional evolution, while in reality physical phenomena take place in higher dimensions.
We describe how similarity solutions can be used to describe singular behavior in higher dimensions, where in general different spatial directions are characterized by different scaling behavior.

Another important issue concerns spatial and temporal complexity. Instead of producing a singularity at a single point, the profile may exhibit a more complex picture through a series of instabilities. This means that instead of the self-similar dynamics tending toward a fixed point, the dynamics tending toward a periodic orbit, corresponding to a sequence of profiles. In an even more complex scenario, the periodic orbit may be replaced by a chaotic dynamics, producing a different instability at every stage. Combining chaotic dynamics with a higher-dimensional description, one obtains a mechanism for spatial complexity, as it is characteristic for turbulent flows.