Introduction to topological fluid dynamics and magnetohydrodynamics

YASUHIDE FUKUMOTO & RENZO RICCA

Part 1 (week 1 - Y. Fukumoto): Basic notions of fluid dynamics and magnetohydrodynamics

Part 2 (week 2 - R. Ricca): Recent progress in topological fluid dynamics

ABSTRACT

Fluid dynamics describes the evolution of fluid flows providing the fundamental laws that govern fluid motion. Magnetohydrodynamics is concerned with the effects that the combined presence of velocity and magnetic fields have on fluid motion. In ideal conditions fluid motion is governed by partial differential equations called Euler equations, that in presence of viscous dissipation become more complex and are known as Navier-Stokes equations. The additional presence of magnetic fields, governed by Maxwell's equations, makes this set of equations even more complete.

Part 1 (week 1) of this course is concerned with a brief derivation of Euler and Navier-Stokes equations from first principles, by focussing on conservation properties associated with vortex motion and helicity. In absence of dissipation topological aspects prevail and hamiltonian techniques provide a useful tool to tackle and solve fundamental problems of ideal fluid motion. In presence of dissipation, however, conserved properties no longer survive, topology changes and energy is gradually eroded. By introducing Maxwell's equations we can consider the influence of magnetic fields and magnetic helicity as well.

Part 2 (week 2) of this course deals with recent developments in topological fluid mechanics, by focussing, in particular, on the presence of coherent structures such as vortex filaments and magnetic flux tubes. In this context helicity admits topological interpretation in terms of linking numbers. Existence of vortex knots and links, recently established both experimentally and analytically, is briefly discussed. Relations between braided magnetic fields and energy contents are derived, and groundstate magnetic energy spectra are discussed in relation to topological complexity. In presence of dissipation reconnections take place and topology changes. We show how this influences helicity, structural complexity and ultimately energy.

Cont. …

COURSE CONTENTS

Part 1: 4-9 September, 2017

Basic notions of fluid dynamics and magnetohydrodynamics

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1. Derivation of Euler and Navier Stokes equations
2. Helmholtz's conservation laws, Kelvin's circulation theorem and helicity
3. Vortex dynamics, Kelvin-Arnold's theorem and hamiltonian techniques
4. Role of viscosity and change of topology: the Burgers vortex
5. Maxwell's equations of magnetohydrodynamics
6. Magnetic helicity, magneto-rotational instability and bifurcations

References

Chorin, A.J. and Marsden, J.E. *A Mathematical Introduction to Fluid Mechanics*. Springer-Verlag, 1993.
Saffman, P.G. *Vortex Dynamics*. Cambridge University Press, 1992.
Davidson, P.A. *An Introduction to Magnetohydrodynamics*. Cambridge University Press, 2001.

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Part 2: 11-16 September, 2017

Recent progress in topological fluid dynamics

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1. Helicity and linking numbers
2. Vortex knots and links: experiments and analytical results
3. Magnetic braids on the Sun
4. Groundstate energy spectra of magnetic knots and links
5. Topological changes under reconnection
6. Knot polynomials and structural complexity

References

Arnold, V.I. and Khesin, B.A. *Topological Methods in Hydrodynamics*. Springer-Verlag, 2013.
Ricca, R.L. (Editor) *Lectures on Topological Fluid Mechanics*. Springer-Verlag, 2009.