Universality of height fluctuations in non-integrable dimer models

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Dimer systems are simplified models of dense liquids of anisotropic molecules. In two dimensions, at close packing, they are closely connected with the Ising model, which they provide an alternative formulation of, and they can be interpreted as models of discrete random surfaces via the notion of height function. If the probability distribution of dimer configurations is associated with a translationally invariant product measure, the model is exactly solvable: the partition function and the correlation functions can be expressed as determinants of the so-called Kasteleyn's matrix and of its inverse, respectively. This allows one to prove that, for an open set of dimer probability weights, dimer-dimer correlations decay polynomially, with a universal critical exponents (independent of the dimer weights), and the fluctuations of the height function scale at large distances to a Gaussian Free Field (GFF), with a universal "elastic constant" (independent, again, of the dimer weights). How much do these properties depend upon integrability? If the dimer distribution is not assumed to be a product measure, how do the critical exponents and height fluctuations change?

In this series of lectures I will discuss a class of non-integrable, "weakly interacting", dimer models. I will first state a result on the large distance behavior of the dimer-dimer correlation, which is characterized by an "anomalous critical exponent", continuously depending upon the strength of the interaction, and then a second one on the fluctuation properties of the height function, which scales to a GFF with a "dressed elastic constant", depending upon the strength of the interaction, as well. Moreover, I will show that the anomalous critical exponent and the dressed elastic constant are related by a universal scaling relation, earlier predicted on the basis of formal bosonization methods. The proof is based on a representation of the generating function of dimer correlations in terms of Grassmann integral, which can be analyzed via rigorous multiscale methods. I will give an introduction to these techniques and describe the main ideas involved in the computation of the scaling limit of dimer correlations and height fluctuations. Lectures based on a series of joint works with Fabio Toninelli and Vieri Mastropietro (partly also with Bruno Renzi).