Geometric dynamical systems methods for multi-scale phenomena, with applications in physics, chemistry, and biology

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In this course, we will study the geometric theory for systems of differential equations in which the solutions exhibit dynamics on multiple scales, including fast-slow systems and systems with multiple length scales. The theory of normally hyperbolic invariant manifolds and center manifolds plays a central role in decomposing the multi-scale phenomena into its different components, and hence in analyzing all aspects of the transient and long-term system dynamics. In addition, modern aspects of the theory of geometric desingularization will be presented, which enable one to study the various singularities that arise in these systems, and hence to analyze the dynamics near sets where normal hyperbolicity is lost. Such singularities, including folded nodes, folded saddles, folded saddle-nodes, and folded toral singularities, lie at the heart of many interesting phenomena including limit cycle canards, mixed-mode oscillations, firing thresholds, and torus canards that mediate the transitions between different rhythms. The theory will be motivated by, and presented jointly with, examples of systems arising in a wide variety of scientific problems, including reaction-diffusion equations, model reduction theory, nonlinear mechanical oscillators, neuroscience, enzyme kinetics, and combustion, and it is planned that the examples will be tailored to student interest. Most importantly, connections will be made to the theory of pattern formation presented in the course by Professor Doelman.