Spectral stability of nonlinear waves: point spectrum and bifurcations

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For dissipative and dispersive problems, it is known that instabilities for nonlinear waves when the associate linearized operator has point spectra in the right-half of the complex plane. Otherwise, the wave is spectrally stable and under certain additional assumptions it can be shown to be nonlinearly stable to classes of small perturbations. While the essential spectrum is relatively straightforward to calculate for localized waves; the point spectra is often challenging to localize analytically. To this end we present an overview of the Evans function – a function which is analytic in the spectral parameter, and whose zeros correspond to point eigenvalues for the associated linear operator. We construct the Evans function and show how to analytically and/or numerically find its zeros. Several applications will be presented, including an especially intriguing application of the Evans function to detect edge bifurcations, i.e., eigenvalues which emerge from the essential spectrum as parameters are varied.