Long nonlinear surface and internal waves on currents: from 2D to 3D

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The Korteweg - de Vries (KdV) equation and its generalisations are successfully used to describe long weakly nonlinear surface and internal waves that are commonly observed in the oceans. The waves described by these models have plane or nearly plane fronts. However, waves registered by satellite observations often have curvilinear fronts and propagate over various currents, motivating the studies of three-dimensional waves. In this short course I will give an Introduction to the recent developments in this area within the scope of the reduced models derived from the full set of Euler equations for a stratified fluid with the boundary conditions appropriate for oceanographic applications. The emphasis will be on surface and internal ring waves propagating over a vertically sheared parallel current, as well as the effects of rotation on the propagation of plane waves. The phenomenology described by the models is rich and sometimes counter intuitive. For example, surface ring waves in a two-layer fluid propagating over a piecewise-constant current have elongated fronts, while the wavefronts of interfacial waves propagating over the same current are squeezed. I will give an explanation to this phenomenon.

Contents:
Lecture 1. Overview of KdV equation as a model of long plane surface and internal waves in stratified fluids. Solitons, Inverse Scattering Transform.
Lecture 2. Gardner and extended KdV equations. Near-identity transformations and asymptotic integrability.
Lecture 3. Ostrovsky equation and effects of rotation. Zero-mass contradiction.
Lecture 4. Far-field modal decomposition and 2+1-dimensional cKdV-type equation for long ring waves on a parallel shear current.
Lecture 5. Singular solutions of nonlinear first-order ODEs and wavefronts of ring waves.
Lecture 6. Application to modelling of long surface and internal ring waves in layered fluids with parallel shear currents.