Non-stationary Navier-Stokes equations: weak solutions and an introduction to related problems

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Abstract

After recalling the results concerning the existence of global in time regular solutions and weak solutions to the non stationary 3D-Navier-Stokes equations, we consider some open problems. In particular for a weak solution the purpose is to draw the attention on the attempts made to analyze and to solve the questions connected with the regularity, the uniqueness and the energy inequality. This will be made either considering well known results and some new result. Since we cannot be exhaustive at all, and to avoid some technical difficulties, we essentially consider the mathematical questions for the Cauchy problem.

References

- For mathematical basic tools for the Navier-Stokes equations it is suggested the monograph:

[1] G.P.Galdi *An introduction to the mathematical theory of the Navier-Stokes equations,* vol.I Springer.

- For the existence of regular and of weak solutions to the non stationary Navier-Stokes equations, one of the following classical monographs is suggested:

[2] P.Constantin and C. Foias, *Navier-Stokes equations*, Chicago Lectures in Mathematics. University of Chicago Press, Chicago, (1988).

[3] O.A. Ladyzhenskaya, *The mathematical theory of viscous incomepressible fluids*, Gordon Breach (1969).

[4] R. Temam, *Navier-Stokes equations*, North-Holland (1977).

- A special attention will be given to the results of papers:

[5] L. Caffarelli, R. Kohn and L. Nirenberg, Partial regularity of suitable weak solutionsof the Navier-Stokes equations, Commun. Pure Appl. Math, **35** (1982).

[6] T. Kato, *Strong L^p-solutions of the Navier-Stokes equation in $\mathbb{R}^m$, with applications to weak solutions*, Math. Z. **187** (1984).