A PDE approach to mean-field disordered systems

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The goal of statistical mechanics is to describe the large-scale behavior of collections of simple elements, often called spins, that interact through locally simple rules and are influenced by some amount of noise. We will focus on the situation where the local interactions are chosen at random, in which case the models are usually called "spin glasses". Such models are already surprisingly difficult to analyze when all spins interact with each other. In this course, we will revisit this analysis using tools from the theory of Hamilton-Jacobi equations. After an overview of the material covered in the course, we will start by discussing the very simple Curie-Weiss model. We will introduce analytical techniques related to the study of Hamilton-Jacobi equations and use them to identify the limit free energy of the model. We will next transpose this strategy into a first-class of disordered models coming from statistical inference. In terms of difficulty, these models are a useful bridge between the Curie-Weiss model and spin glasses. We will finally turn our attention to the latter models, for which infinite-dimensional Hamilton-Jacobi equations arise.