Normal form methods with numerical applications

The lectures will be devoted to the study of dynamics in a neighbourhood of an equilibrium point of differential equations, with particular emphasis on the case of a Hamiltonian system in a neighbourhood of an elliptic equilibrium.

The aim is to discuss at the same time the analytical methods and the numerical applications to the problem of stability.

Specific topics will cover:

i. Construction of first integrals for the Hamiltonian case, with quantitative estimates leading to exponential stability in Nekhoroshev's sense.

ii. Normal form methods based on the use of Lie series and Lie transform. The case of convergent series (Poincaré–Siegel center problem, Kolmogorov's theorem) and of exponential stability.

iii. Use of computer algebra in order to perform explicit expansions. Computer-assisted study of long time stability for specific models.

iv. Applications to the study of long time stability for realistic models, such as: the Lagrangian equilibria of the restricted problem of three bodies; the Lagrange theory of secular motions of the planets; the neighbourhood of an invariant Kolmogorov torus.

Some applications to simple models will be suggested as exercises to be worked out in groups. We plan to assure some assistance.