Mathematical billiards: ergodicity and renormalization, from polygons to the Ehrenfest model

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Mathematical billiards are an idealisation of the billiard game in differently shaped "tables", which arise naturally from the study of several problems in physics. Famous planar billiards include the Lorentz gas and the Ehrenfest model (Figure 2), introduced at the beginning of 1900.

In this course we will focus on billiards in (planar) *polygonal* tables (*bounded*, as in Fig. 1, or *unbounded* but *periodic*, as in Fig. 2) and on the question of *ergodicity*, which is one of the fundamental features studied in a chaotic system since the Boltzmann ergodic hypothesis.

We will sketch the proof of some classical results in the study of rational polygonal billiards, as well as some very recent breakthroughs in our understanding of chaotic properties of infinite polygonal billiards such as the Ehrenfest model. A key idea, which has been exploited successfully for several decades in the study of rational polygonal billiards, is to *unfold* them into surfaces. This allows to use powerful renormalization tools exploiting dynamics in a space of geometric structures. The course will provide a self-contained introduction to the beautiful ideas behind this approach.

Tentative syllabus:

- Motivation to study mathematical billiards: some examples from physics;
- Basic definitions: billiard flow, basic dynamical systems and ergodic theory notions;
- Unfolding a rational polygonal billiard to a surface; the space of flat surfaces;
- Renormalization in the space of flat surfaces: Masur's criterium for ergodicity;
- Examples of unfolding to infinite periodic flat surfaces (see Figure 3): the retroreflector, the Ehrenfest billiard and systems of Eaton lenses;
- Survey of some recent results on the Ehrenfest model and Eaton lenses; some ideas of the proof of superdiffusion and of the non-ergodicity results.

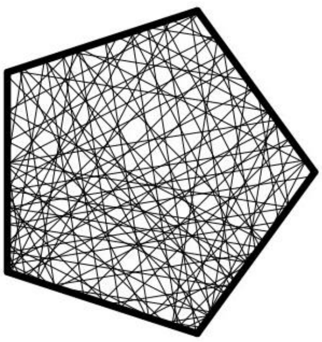

Figure 1. Pentagonal billiard

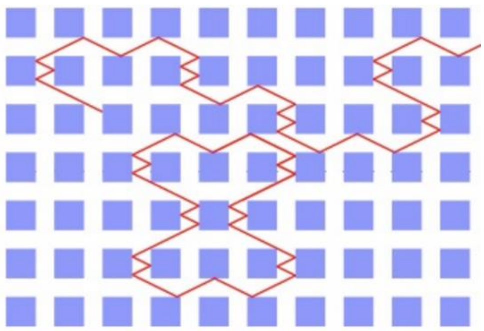

Figure 2. The Ehrenfest Windtree Model

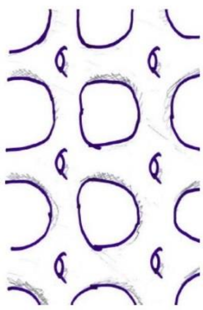

Figure 3. Periodic surface

Some bibliographic references

H. Masur and E. Tabachnikov, Rational billiards and flat structures, *Handbook of dynamical systems*, Vol. 1A

Carlos Matheus, Diffusion in the Ehrenfest windtree model, *Disquisitiones Mathematicae*

Fraczek, K & Ulcigrai, C, 2014, 'Non-ergodic Z-periodic billiards and infinite translation surfaces'. *Inventiones Mathematicae*, 2014