Numerical methods for balance laws

Giovanni Russo (Catania)

Abstract:
The course constitutes an introduction to the construction and application of effective method for the numerical solution of systems of balance laws.
Starting from simple three point schemes for linear advection equation, the course will give a glimpse on the construction of high order finite volume and finite difference shock capturing schemes for the numerical solution of quasilinear systems of conservation and balance laws, including the numerical treatment of relaxation systems with stiff source.
Emphasis will be given on the ideas behind the various methods, with practical examples illustrating how to implement some of the schemes on selected models.

The list of topics included in the course is the following:

1. Three point methods for the scalar equation: upwind, Lax-Friedrichs and Lax-Wjndroff. Generalization to linear hyperbolic systems.
2. Shock capturing finite volume methods for conservation laws based on Riemann solver: Godunov method and relation with first order upwind method for linear systems.
3. General structure of semi discrete conservative finite volume shock capturing schemes for systems of conservation laws: numerical flux function, non-linear reconstruction, time integrator. High order finite difference schemes.
4. Space discretization: high order essentially non oscillator reconstructions (ENO and WENO). Time discretization: Strongly Stability Preserving Runge-Kutta schemes.
5. Treatment of stiff source. The concept of Asymptotic Preserving schemes for hyperbolic to hyperbolic relaxation. Implicit-Explicit Euler scheme. High order time discretization: Implicit-Explicit Runge-Kutta schemes.
6. Generalizations: high order semi-implicit schemes, with application to a variety of pdes including non-linear diffusion equations, and/or penalisation methods for the construction of high order finite volume schemes for relaxation systems, and for the time dependent Boltzmann equation for small, but non negligible, Knudsen number (mention).