The dimer model: Glauber dynamic and mixing times

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Glauber dynamics are stochastic processes (more precisely Markov chains) that mimic the time evolution of statistical physics systems. Alternately, theoretical computer scientists view them as probabilistic algorithms that simulate the Boltzmann-Gibbs distribution of a model of interest. A natural question is how fast (as a function, say, of the system size) the Markov chain converges to its equilibrium distribution. This can be measured for instance via the spectral gap or by the mixing time. The speed of convergence is deeply related with the equilibrium properties of the model, slow spatial mixing of correlations usually resulting in slow (in time) convergence to equilibrium.

I will first introduce some basic definitions and tools for Markov chain mixing (total variation distance, spectral gap, path coupling, monotonicity...) and then I will specialize to the case of the Glauber dynamic of the two-dimensional, planar dimer model (that will be introduced by A. Giuliani in his series of lectures in the first week, but I will recall the basic notions). The dimer model dynamic is a particularly interesting example since it connects both to interacting particle systems and to dynamics of 2-dimensional discrete surfaces and of planar tilings.

Some parts of these lectures are based on a series of joint works with Benoit Laslier, Fabio Martinelli and Pietro Caputo.