Incremental Equations of Instability for Soft Solids

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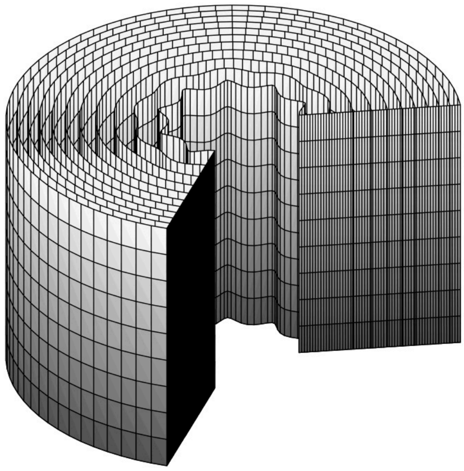
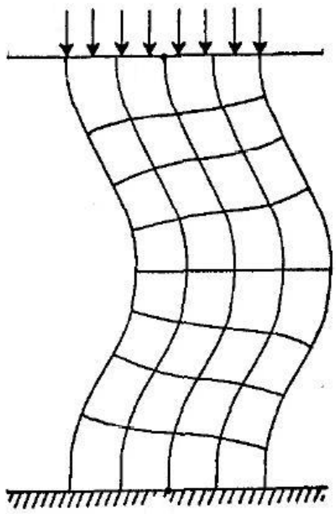
Summary: The theory of non-linear elasticity provides a wonderful framework to write down the equations governing the onset of small-amplitude wrinkles on the surface of highly deformed soft solids. Specifically, we can focus on isotropic incompressible models, such as those used to describe the behaviour of rubbers, silicones and gels. For this class of solids there exist universal solutions of practical interest, such as simple extension, simple shear, torsion or bending. By linearising the equations of equilibrium in the neighbourhood of one of these large deformations, we obtain the so-called incremental equations of elasticity. If we can find a solution to those, then we can argue that a threshold of instability has been reached by the large deformation, because it now admits adjacent states of equilibrium. Here we will obtain bifurcation criteria for large homogeneous deformations (and, time permitting, for large bending) and link them to experimental observations of wrinkles.

Proposed Syllabus

1. Introduction
   1.1. Euler stability criterion
   1.2. Equilibrium of large deformations
   1.3. Increments
   1.4. Instantaneous moduli



2. Internal stability
   2.1. Strong ellipticity condition
   2.2. Example: Mooney-Rivlin materials (rubber, gels)
   2.3. Example: Varga materials (rubber)
   2.4. Example: Fung materials (soft tissues)
3. Surface (Biot) stability
   3.1. Boundary value problem
   3.2. Stroh formulation
   3.3. Examples: plain strain, simple shear
4. Slab instability
   4.1. Boundary conditions
   4.2. Bifurcation criterion
   4.3. Link with classical results of Euler buckling
   4.4. Examples



5. Bending instability
   5.1. Pure flexure of a slab
   5.2. Stroh formulation
   5.3. Numerics: Compound matrix method
   5.4. Examples