Dynamics of defects and fronts in pattern-forming systems

Part 1: Local bifurcations and existence of defects

During the last decades the use of bifurcation theory, led to significant progress in the understanding of nonlinear phenomena in partial differential equations, including hydrodynamic problems, structural mechanics, but also pattern formation, or population dynamics. In the first part of these lectures we present two specific methods that arise in the analysis of local bifurcations in infinite-dimensional systems, namely the center manifold reduction and the normal form theory. We then show how these methods are used in the study of defects arising in spatially periodic patterns. We focus on the existence of dislocations and grain boundaries in the Swift-Hohenberg equation, a prototypical model system which is extensively used in pattern formation theory.

Part 2: Pattern-forming fronts

We study patterns that arise in the wake of propagating fronts. Examples are phase separation processes, such as the famous Liesegang experiment, fluid instabilities, or manufacturing processes such as ion beam milling. As prototypical model is the Cahn-Hilliard equation with a uniformly translating source of mass deposition. We will study questions such as the existence of fronts, stability and selection questions, and more subtle front interaction problems.