Problem E3. What can you say about $\| \mathcal{A} - \mathcal{A}_r \|_F$ assuming that $\sigma_{\tilde{r}_1}(A_{(1)}) \leq \delta$, $\sigma_{\tilde{r}_2}(A_{(2)}) \leq \delta$, and $\sigma_{\tilde{r}_3}(A_{(3)}) \leq \delta$?

Problem A3. In the QR with column pivoting (QRP) decomposition $AP = QR$ the upper triangular matrix $R \in \mathbb{R}^{n \times n}$ is graded in the sense that

$$r_{jj}^2 \geq \sum_{i=j}^{k} r_{ik}^2 \qquad k = j{:}n$$

Formulate an HOQRP factorization for a tensor $\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ that is based on the QR-with-column-pivoting factorizations

$$\mathcal{A}_{(k)} P_k = Q_k R_k$$

for $k = 1{:}3$. Does the core tensor have any special "grading" properties?