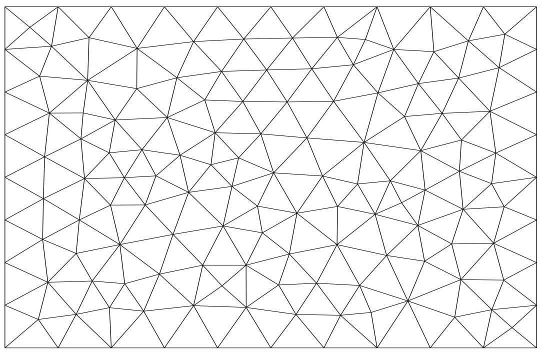
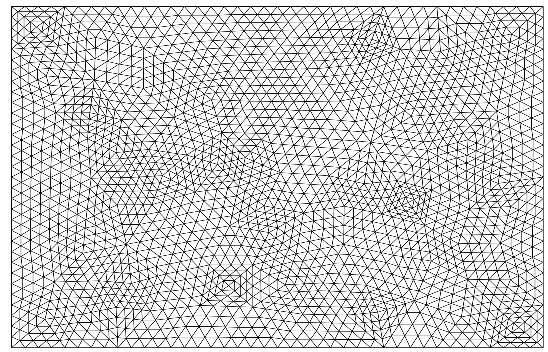
Il metodo agli Elementi Finiti. “Triangolazione” del Dominio



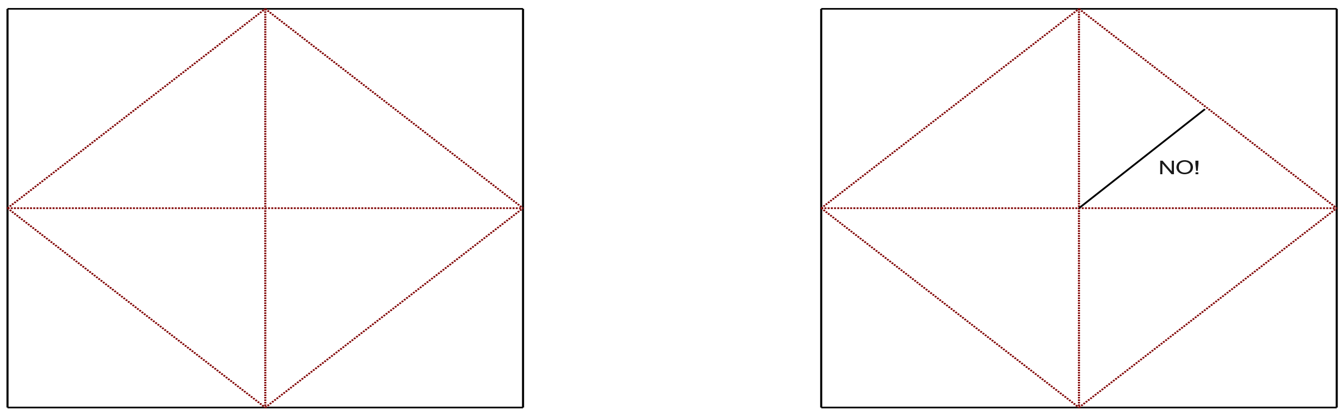


Δ_i: i-esimo triangolo

$$\Omega_h = \cup_{i=1}^{m} \Delta_i \quad \approx \quad \bar{\Omega}$$

Il metodo agli Elementi Finiti. “Triangolazione” del Dominio

Fatto di base:



dimensione della griglia (mesh): $h = \max_{i=1,m} \mathrm{diam}(\Delta_i)$

(diametro=lunghezza del lato più lungo)

Il metodo agli Elementi Finiti. Spazio dei polinomi “a tratti”

$$S_h = \{\varphi \ t.c. \ \varphi|_{\Omega_h} \text{ continua}, \ \varphi|_{\partial\Omega_h} = 0, \ \varphi|_{\Delta_j} \text{lineare} \, \forall j\}$$

Polinomi *lineari* a tratti (polinomi compositi)

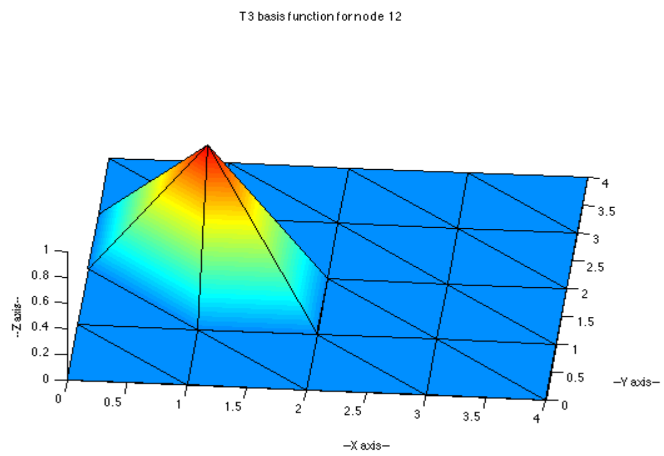
Per ogni nodo $\mathbf{x}_j$, $j = 1, \ldots, n$, ad ogni funzione $v_h \in S_{h,0}$ è possibile associare in modo univoco una funzione φ_j:

$$\varphi_j(\mathbf{x}_i) = \delta_{i,j} = \begin{cases} 1 & se \quad i = j \\ 0 & se \quad i \neq j \end{cases}$$

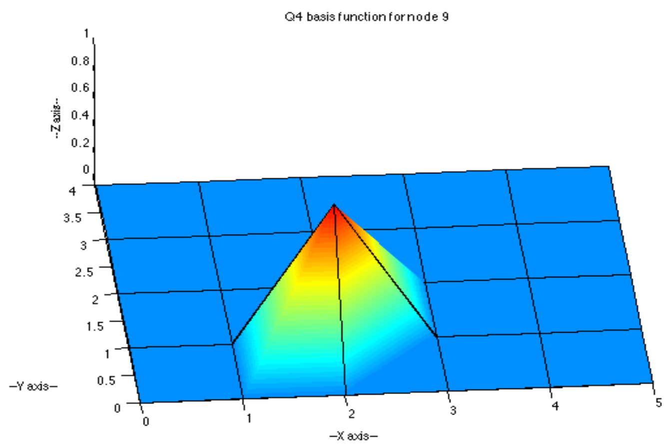
$$v_h \in S_{h,0}, \quad v_h(\mathbf{x}) = \sum_{j=1}^{n} \alpha_j \varphi_j(\mathbf{x})$$

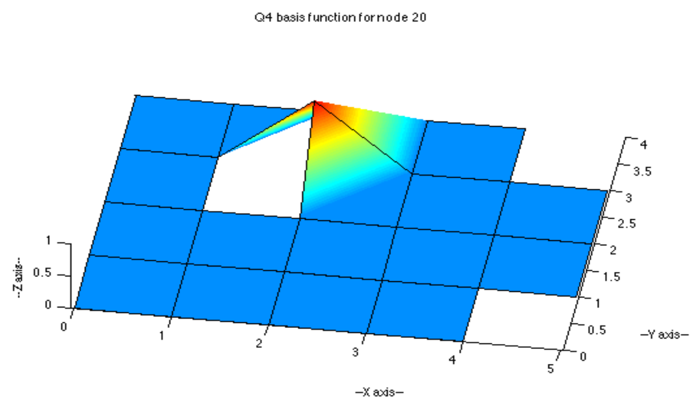
Il metodo agli Elementi Finiti. Funzioni di base

$$\varphi_j(\mathbf{x}_i) = \delta_{i,j} = \begin{cases} 1 & se \quad i = j \\ 0 & se \quad i \neq j \end{cases}$$

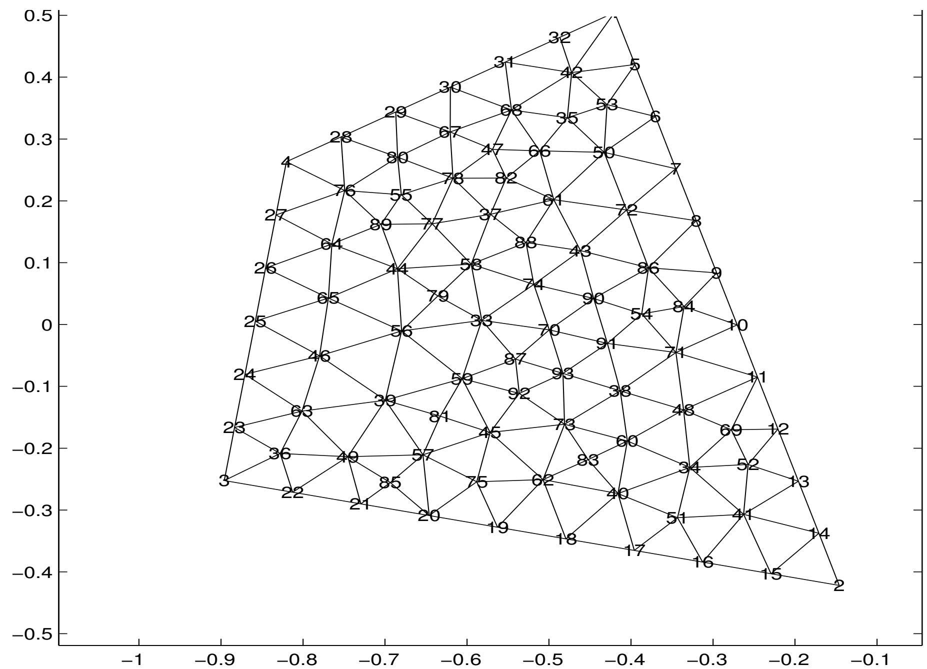


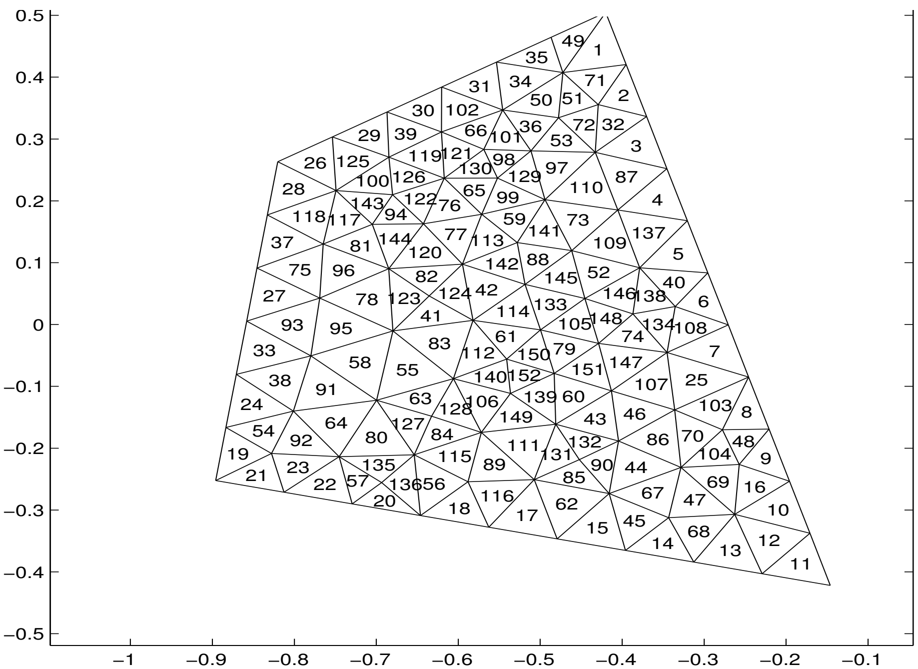
Elementi su rettangoli

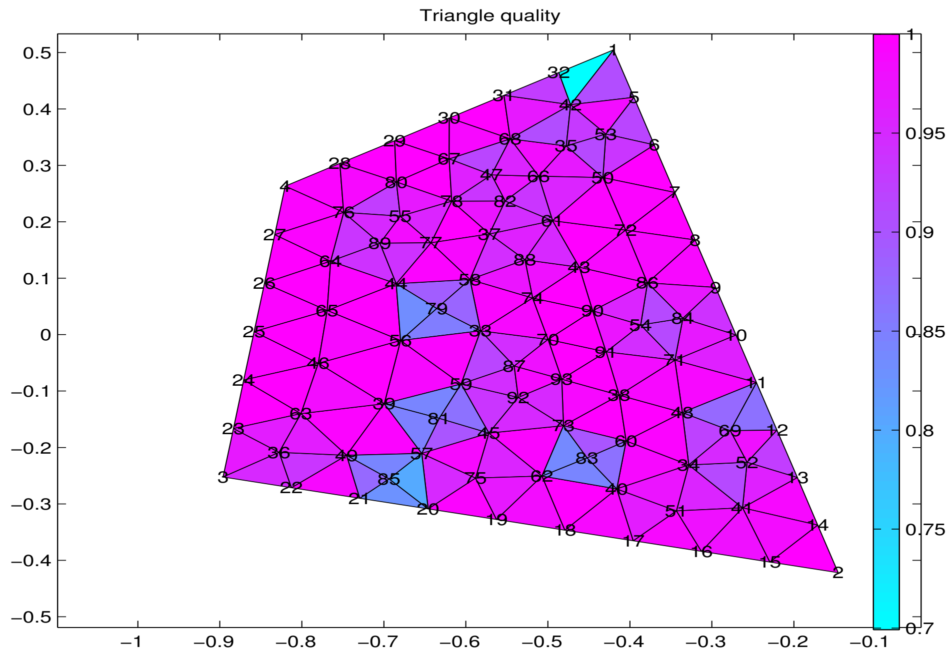




Numerazione e bontà della griglia


0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−1
−0.9
−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1


0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−1
−0.9
−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1



Triangle quality

Assemblaggio

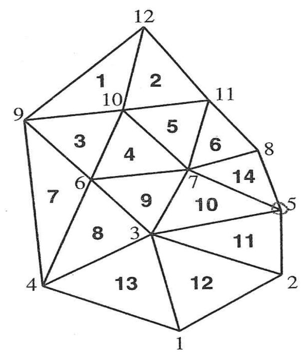


FIG. 1.17. Nodal and element numbering for the mesh in Figure 1.5.

Figure 1.17 we introduce the connectivity matrix defined by

$$
\begin{array}{c}
\begin{array}{cccccccccccccc}
\mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14}
\end{array}\\
\mathtt{P}^T = \begin{bmatrix}
9 & 12 & 9 & 6 & 10 & 11 & 4 & 4 & 6 & 5 & 5 & 2 & 1 & 8 \\
10 & 10 & 6 & 7 & 7 & 7 & 6 & 3 & 3 & 7 & 3 & 3 & 3 & 7 \\
12 & 11 & 10 & 10 & 11 & 8 & 9 & 6 & 7 & 3 & 2 & 1 & 4 & 5
\end{bmatrix},
\end{array}
$$

so that the index $\mathtt{j} = \mathtt{P}(\mathtt{k}, \mathtt{i})$ specifies the global node number of local node $\mathtt{i}$ in element $\mathtt{k}$, and thus identifies the coefficient $\mathbf{u}_i^{(k)}$ in (1.31) with the global coefficient $\mathbf{u}_j$ in the expansion (1.18) of u_h. Given $\mathtt{P}$, the matrices A_k and vectors $\mathbf{f}_k$ for the mesh in Figure 1.17 can be assembled into the Galerkin system matrix and vector using a set of nested loops.

```
k = 1:14
  j = 1:3
    i = 1:3
      Agal(P(k,i),P(k,j)) = Agal(P(k,i),P(k,j)) + A(k,i,j)
    endloop i
    fgal(P(k,j)) = fgal(P(k,j)) + f(k,j)
  endloop j
endloop k
```

A few observations are appropriate here. First, in a practical implementation, the Galerkin matrix Agal will be stored in an appropriate sparse format. Second, it should be apparent that as the elements are assembled in order above, then for any node s say, a stage will be reached when subsequent assemblies do not

Griglia raffinata intorno a singolarità. Rettangoli e Triangoli