Numerical Linear Algebra, PhD Course 2021

Computer Laboratory, 16/12/2021

Step 1. FE discretization using PDEToolbox di Matlab.
Open the Toolbox by typing: `pdetool` within Matlab.

1. **Es.1. Poisson equation.**
   (1) In "Option"-"Specification" choose `Generic Scalar` as problem. In "PDE"-"PDE Specifications" choose `Elliptic problem`. Define a polygonal domain (e.g., using the polygon button), and a grid refinement.
   Note: By clicking on the multiple triangle, the grid is refined and more triangles/nodes appear.
   (2) Export `Mesh`, `PDE` coeffs. and `Boundary` data
   (3) Assemble PDE: type `[K,F]=assempde(b,p,e,t,c,a,f);`
   This gives the data for solving the system $Ku = F$
   The problem size depends on the grid refinement. We are interested in analyzing the performance of iterative methods by refining the mesh.
   (4) Type `spy(K)` to explore the sparsity pattern and size of K. Use `p=symamd(K); spy(K(p,p))` to see the reordered entries; do the same for `p=symrcm(K)`.
   (5) For each dimension, use the data `K, F` to run CG with different choices of accelerations:

   ```
   tic;[X,FLAG,RELRES,ITER,RESVEC] = pcg(K,F,1e-8,1000);timeCG=toc;
   ```

   Compute also an estimate of κ(`K`) by means of the Matlab command `condest`. For each dimension display:

   DIM, ITER, RELRES, COND(K)

   For each dimension, plot the residual convergence history (contained in `RESVEC`) in the same figure.

   Comment on the results.

2. **Es.2. Shifted system.**
   Consider the system $(K + \omega i I)u = F$, with K and F as above, and $\omega = 4$.
   (1) Verify that `pcg` does not converge with these data;
   (2) Download the code `pcg1.m` from our website, save it and inspect its implementation; then run `pcg1.m` on the given data and comment on the results. Note: in input set $U = I$ and $L = I$ where I is the identity matrix of the same size as K.