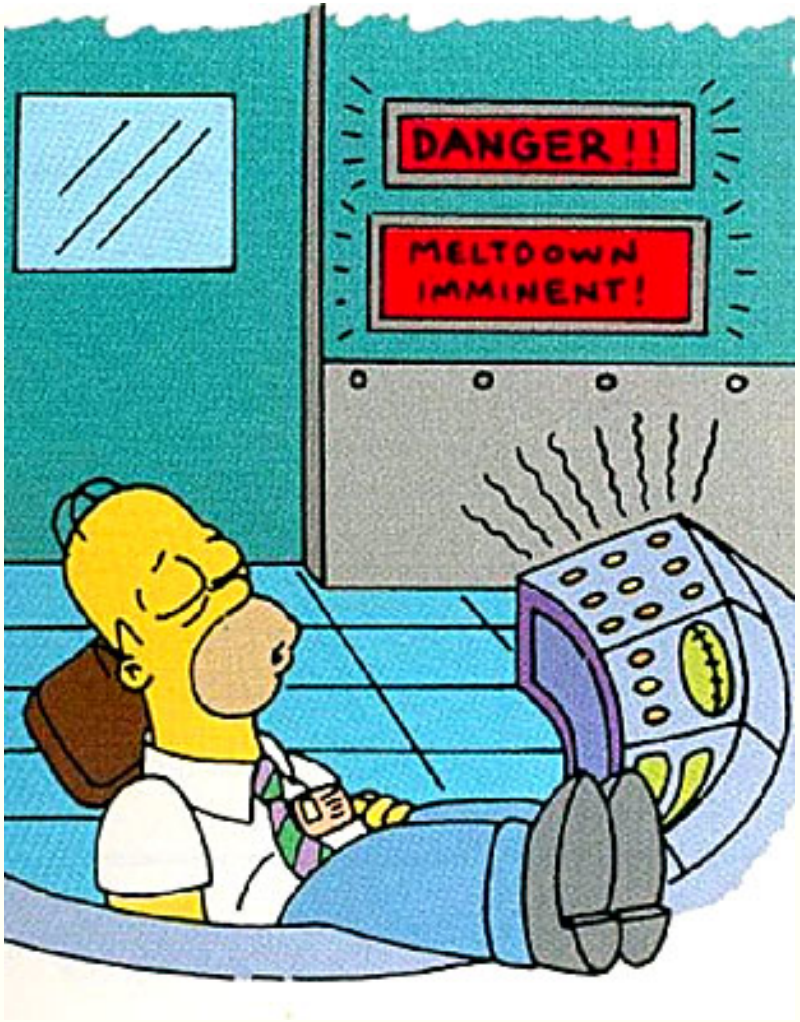
What is the Cost of Falling Asleep During the "Numεrical Analysis and Sciεntific Computing" Class?



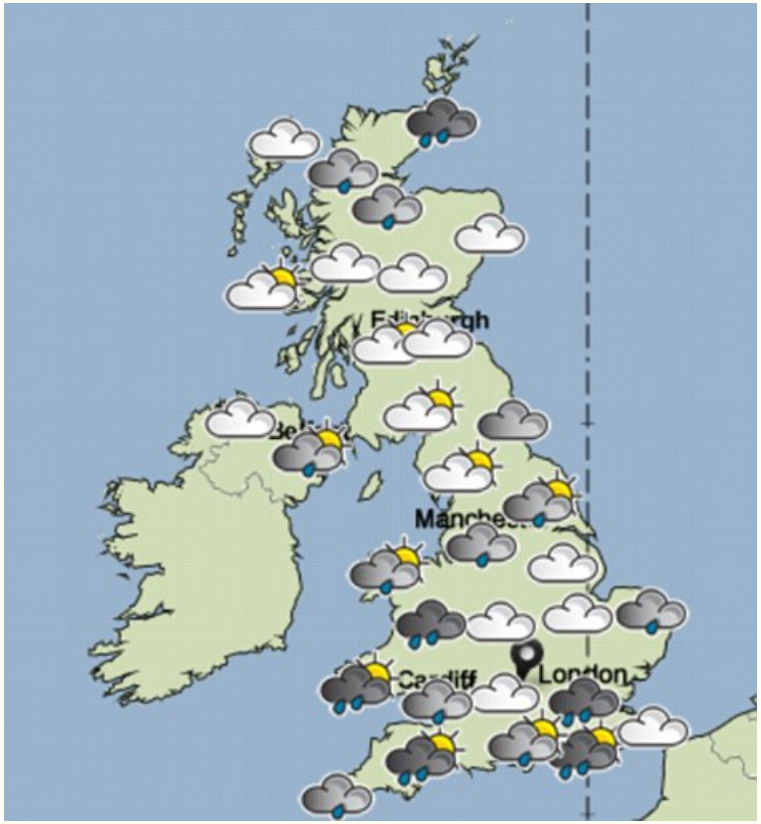
Nataša Strabić
natasa.strabic@manchester.ac.uk

Mathematics Research Students' Conference

24th September 2014

I'm not doing Numerical Analysis, why should I care?

I'm not doing Numerical Analysis, why should I care?



Weather forecasting models require some of the most powerful super-computers in the world.

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To build earthquake proof buildings the issue is to design structures that will not resonate with the induced frequencies, which involves nonlinear eigenvalue problem analysis.

On the left, *Torre Mayor* in Mexico City, Mexico, designed to withstand 8.5 on the Richter Scale.
In January 2003, people inside did not even feel a 7.6 earthquake!

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Eigenvalue analysis is also used to design car stereo systems so that the sound is good for the passengers and driver.

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Numerical linear algebra is essential for financial trading strategies and portfolio optimization.

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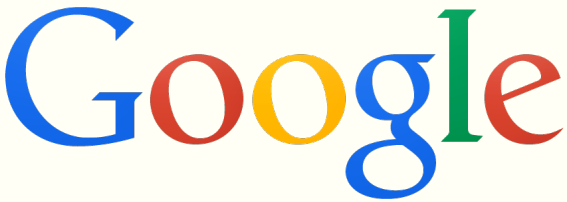
Google PageRank algorithm works with a matrix of size N, where N is the number of webpages on the Internet.



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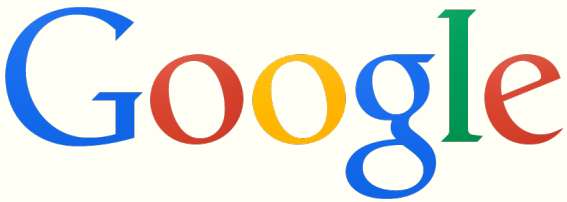

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- The "Inventor of the World Wide Web" is Sir Timothy John Berners-Lee.
- The first website built was at CERN and was first put online on 6 August 1991.
- In September 2014 the number of one billion websites was exceeded.

Outline

Why did it go wrong

The Patriot missile failure

The official report:

"On February 25, 1991, a Patriot missile defense system operating at Dhahran, Saudi Arabia, during Operation Desert Storm failed to track and intercept an incoming Scud. This Scud subsequently hit an Army barracks, killing 28 Americans."

Patriot: The details

- A detected object is tracked by computing the "range gate".
  - This requires information on time.
- The computer had 24 bit fixed point registers.
- The system's internal clock recorded time as the number of tenths of seconds elapsed since last restart.

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chopped at 24 bits, which produces an error of $\approx 9.5 \times 10^{-8}$ seconds.

Patriot: The details (continued)

- The computer had been up around 100 hours.
- The total error in time due to the small chopping error

$$9.5 \times 10^{-8} \times 100 \times 60 \times 60 \times 10 = 0.34 \text{ seconds.}$$

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- The patch was delivered the day after the incident.

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- "Try turning the computer off and back on?"

Ariane 5 Flight 501

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- after 10 years of developement
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On its maiden voyage on June 4, 1996, Ariane 5 rocket produced by the European Space Agency

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- The guidance system's computer tried to convert one piece of data – the horizontal velocity of the rocket with respect to the platform – from a 64-bit floating point number to a 16-bit signed integer.
- The number was larger than 32 768, the largest integer storable in a 16 bit signed integer *(overflow)* and thus the conversion failed.

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The irony

The variable is not needed after take off – this piece of code should have been turned off.

Also, those expensive satellites? They were uninsured.

The Sleipner A platform

- The platform produces oil and gas in the North Sea.
- It is supported on the seabed at a water depth of 82 m.
- The base structure consisting of 24 cells, four are elongated to shafts supporting the platform.

The Sleipner A platform (continued)

- The first concrete base structure for Sleipner A sprang a leak and sank outside Stavanger, Norway on 23 August 1991.
- The crash caused an earthquake registering 3.0 on the Richter scale.
- The failure involved a total economic loss of about $700 million.

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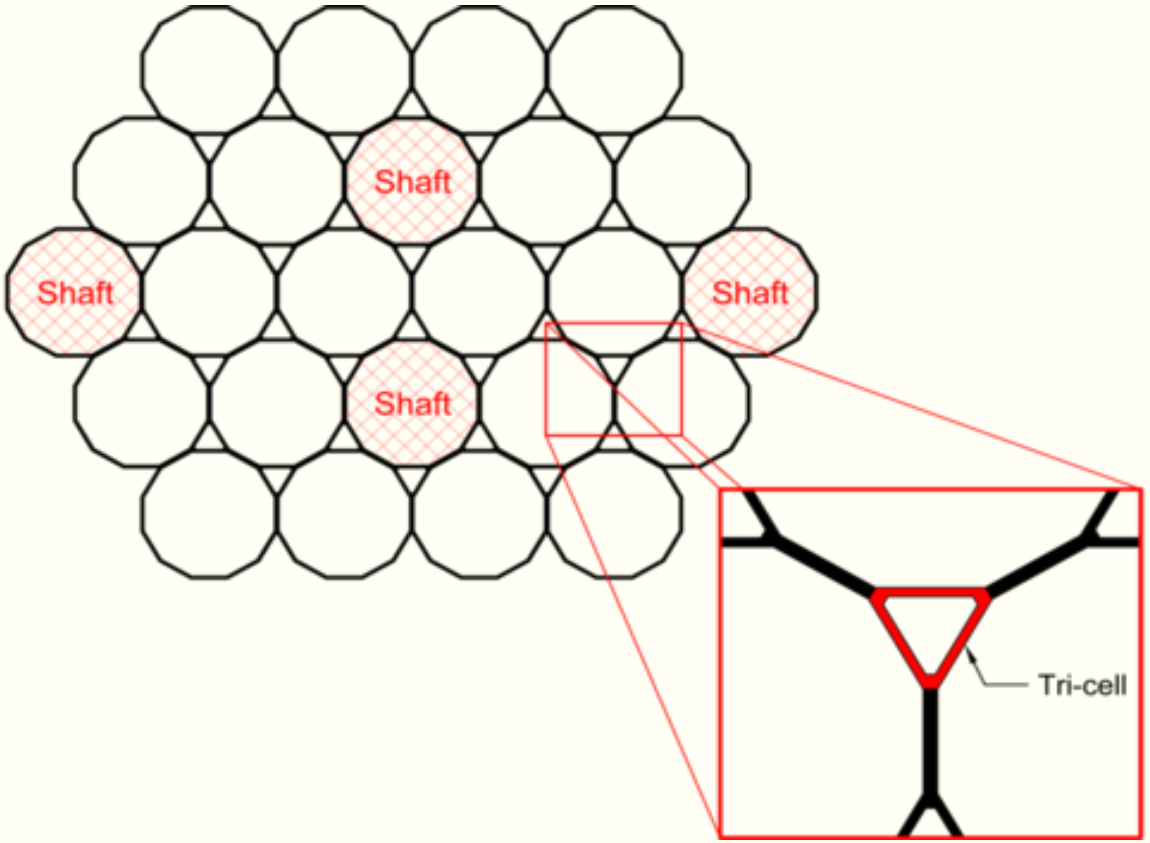

- The cell wall failure was traced to the inaccurate finite element approximation of the linear elastic model of the tri-cell.
- The shear stresses were underestimated by 47%, leading to insufficient design.

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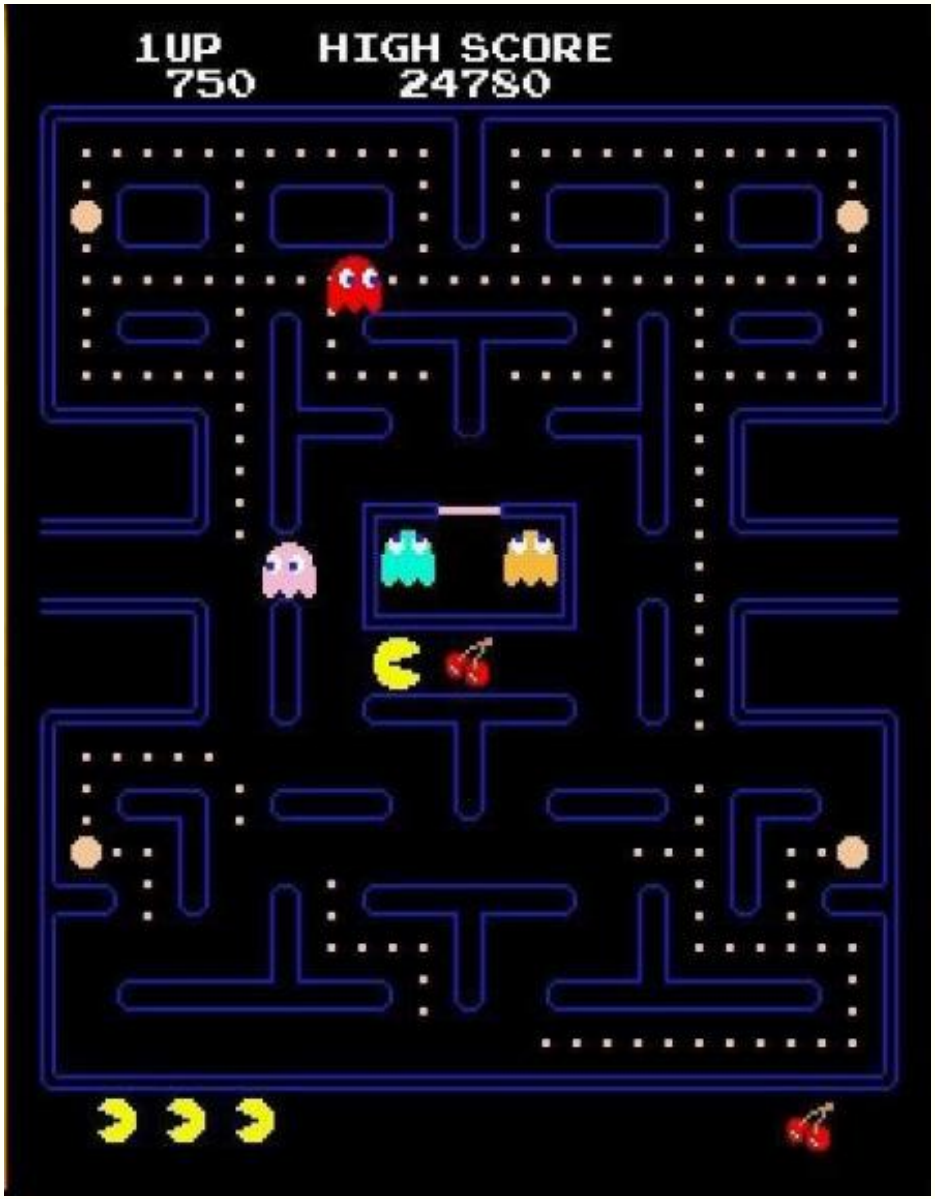

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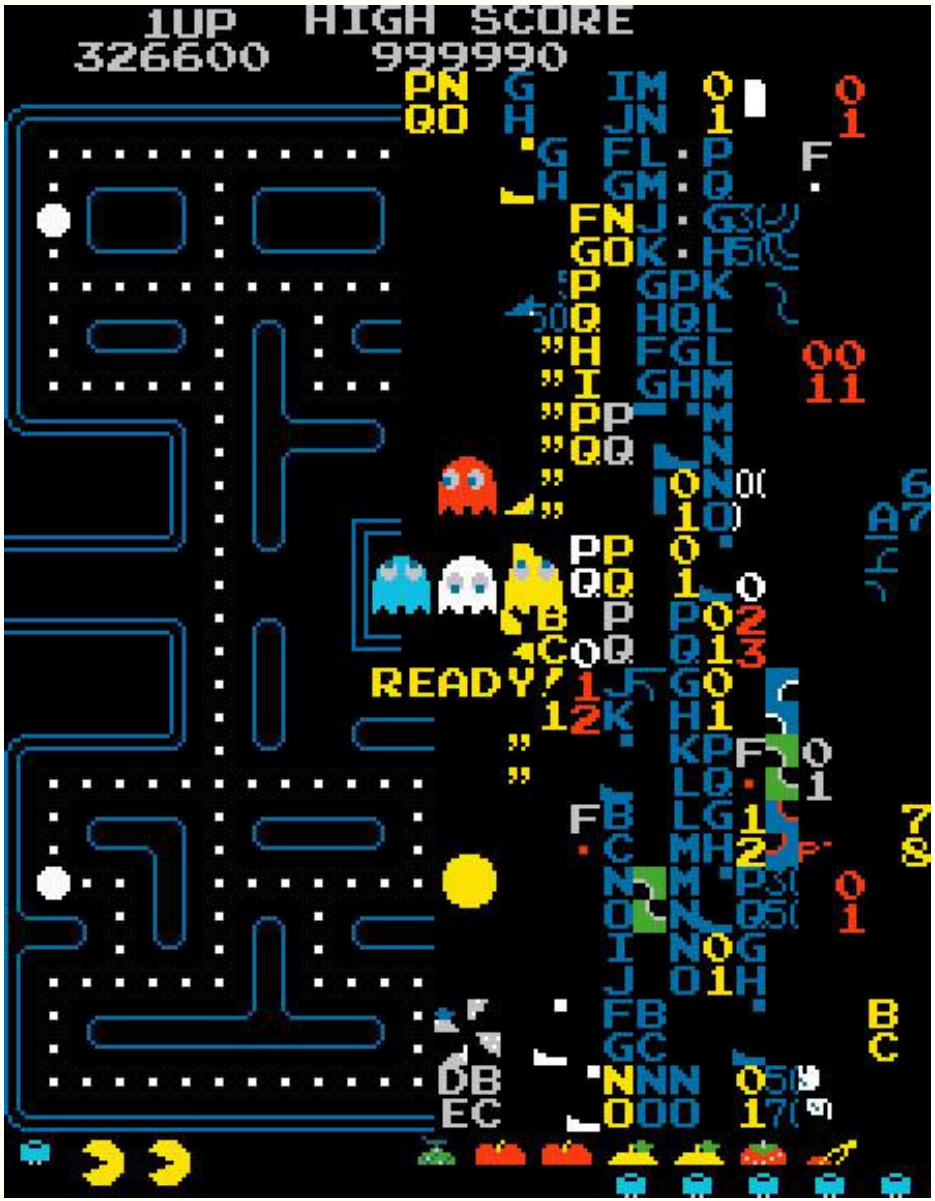

- The game was designed to have no ending, but a bug keeps this from happening.
- On level 256, the right half of the screen gets corrupted making it impossible to eat enough dots to beat the level.

Pac-Man: The details

- Normally, no more than seven fruit are displayed.
- Internal level counter is stored in a single byte (8 bits).
- Once it reaches 255 (maximum representable value), the subroutine that draws the fruit causes *overflow* and tries to draw 256 fruit instead of the usual seven, resulting in a corrupted screen and game over.

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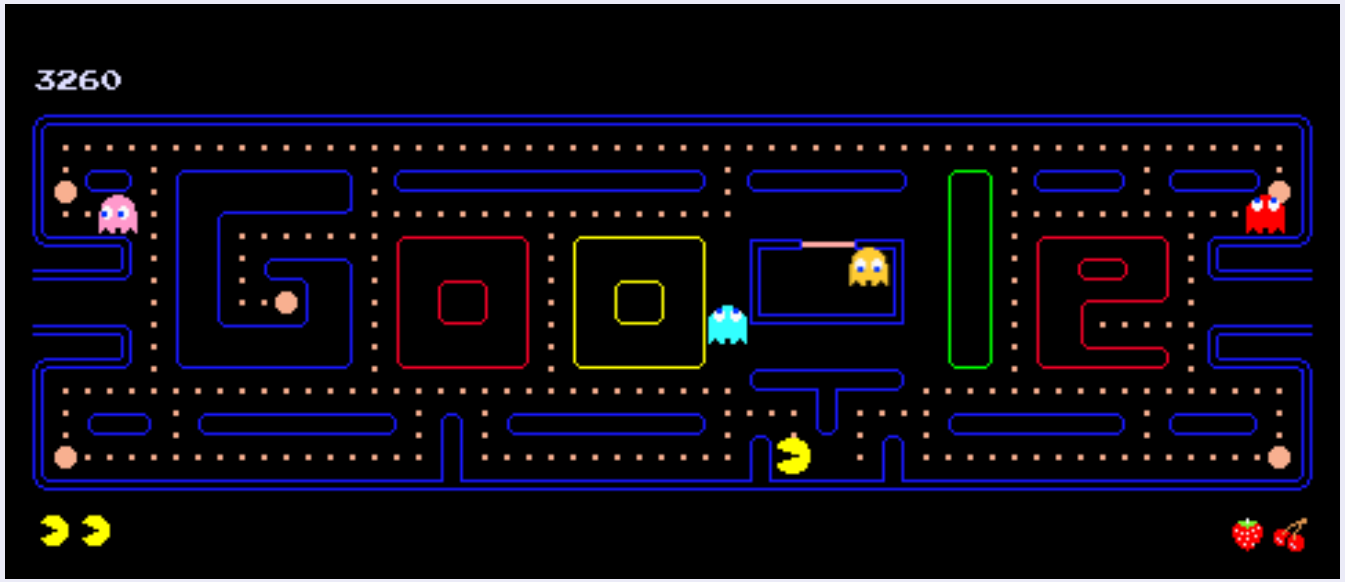
Google Doodle, 30th anniversary of release (2010)



My computer can't do maths

Factorials

For a nonnegative integer n, $\quad n! := 1 \times 2 \times 3 \times \cdots \times n.$

$$5! = 120$$
$$10! = 3628800$$
$$15! = 1307674368000$$
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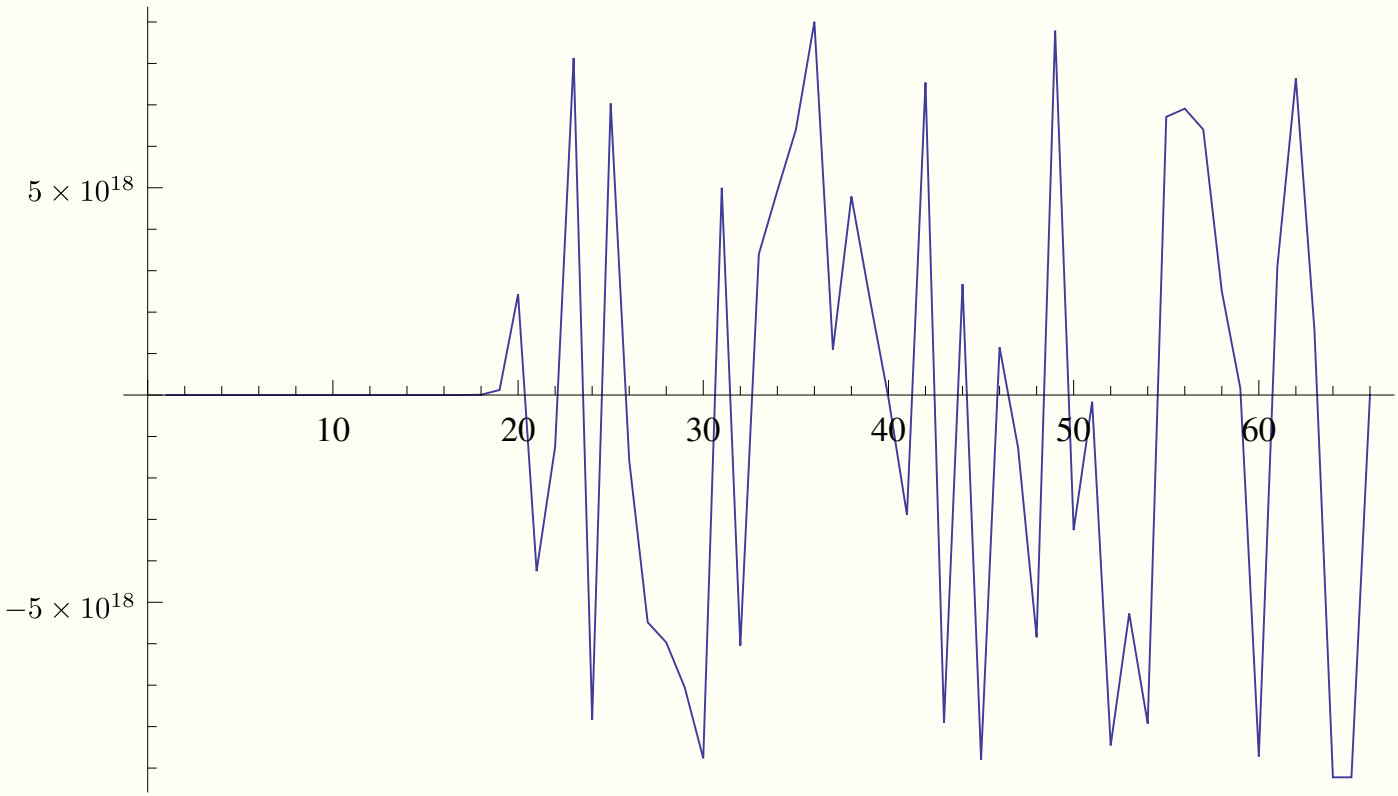
It's Ariane 5 and Pac-Man all over again!

Factorials (continued)

1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320
9! = 362880
10! = 3628800
11! = 39916800
12! = 479001600
13! = 6227020800
14! = 87178291200
15! = 1307674368000
16! = 20922789888000
17! = 355687428096000
18! = 6402373705728000
19! = 121645100408832000
20! = 2432902008176640000
21! = -4249290049419214848
22! = -1250660718674968576
23! = 8128291617894825984
24! = -7835185981329244160
25! = 7034535277573963776
⋮
62! = 7638104968020361216
63! = 1585267068834414592
64! = -9223372036854775808
65! = -9223372036854775808
66! = 0



Calculus

$$\frac{1+\exp(-100)-1}{1+\exp(-100)-1}=$$

$$\left(\frac{1}{\cos\left(100\pi+\pi/4\right)}\right)^2=$$

$$\log\left(\exp(3000)\right)/1000=$$

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$$\left(\left(\cdots\left(\sqrt{\sqrt{\cdots\sqrt{4}}}\right)^2\cdots\right)^2\right)^2 = 1$$

The zero phenomenon

```
[B,C] = meshgrid(linspace(-10,0,128));
G = - B - C + B + C;
contourf(B,C,G);
```

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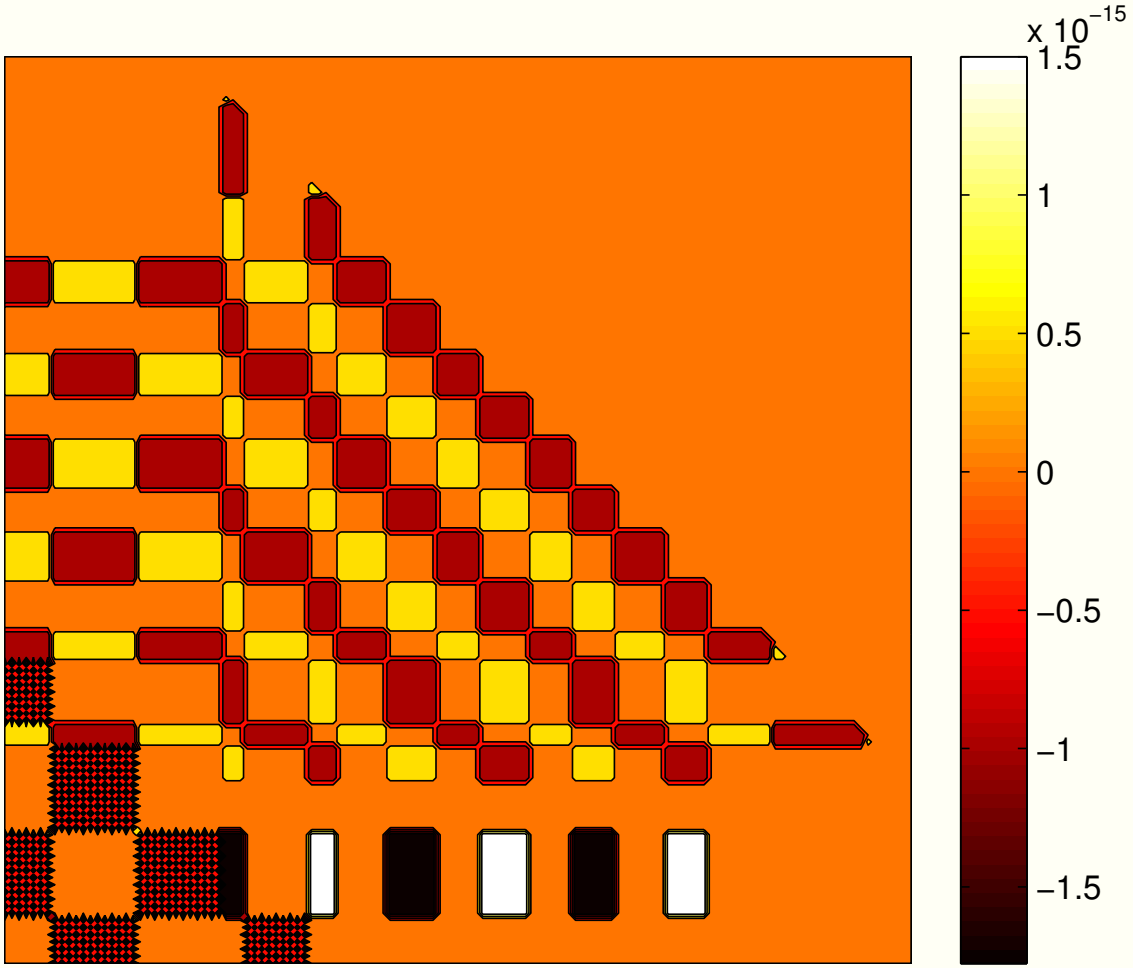

Does tiny stay tiny?

System I.

$$\begin{aligned} 2x &+ 6y &&= 8 \\ 2x &+ 6.0001y &&= 8.0001 \end{aligned}$$

Solution: $x = 1$, $y = 1$.

System II.

$$\begin{aligned} 2x &+ 6y &&= 8 \\ 2x &+ 5.99999y &&= 8.00002 \end{aligned}$$

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- In the 1940s, very pessimistic predictions about the effectiveness of computers for systems of equations.
- Alan Turing and James Wilkinson were the first to understand the correct way to analyze the effects of inexact computer arithmetic *(backward error analysis)* and the solid foundations of NA and NLA of today were set.

Finally, what do Numerical Analysts do?

Design methods that compute approximate solutions to the problems we are not able to solve exactly or fast enough and analyse the resulting errors.

Numerical Analysts make computers solve real-world problems.

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PS. NEVER BLINDLY ASSUME THAT THE COMPUTER IS CORRECT.

References

- The Patriot Missile Failure: link link
- Ariane 5: link link
- Sleipner A platform: link
- Pac-Man: link

[1] Nicholas J. Higham. Accuracy and Stability of Numerical Algorithms. Second edition, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2002.