

ANALISI MATEMATICA T-B
Corso di Laurea in Ingegneria Meccanica, Anno Accademico 2014/15

Esercizi sugli integrali generalizzati

Stabilire se i seguenti integrali sono convergenti:

1. $\int_0^{+\infty} \frac{x^2 + x}{x^4 + 2} dx$ [SI]; **9.** $\int_0^{+\infty} \frac{x^2 + 1}{x + 2} dx$ [NO];

2. $\int_0^{+\infty} \frac{x^3 - x + 1}{x^4 + 2x} dx$ [NO]; **10.** $\int_0^{+\infty} \frac{\sqrt{x^2 + x}}{x + 2} dx$ [NO];

3. $\int_0^2 \frac{x}{2 - x} dx$ [NO]; **11.** $\int_0^1 \frac{\sqrt[3]{x}}{\sqrt{x^2 + x}} dx$ [SI];

4. $\int_1^2 \frac{\sqrt{x - 1}}{\sqrt{x^3 - 1}} dx$ [SI]; **12.** $\int_1^{+\infty} \frac{\sqrt{x^2 + x}}{\sqrt[3]{x^7 + 1}} dx$ [SI];

5. $\int_0^{\frac{\pi}{2}} \frac{x}{\sqrt{\sin x}} dx$ [SI]; **13.** $\int_0^1 \ln x dx$ [SI];

6. $\int_0^{+\infty} \frac{\sin x}{x^4 + x^2 + x} dx$ [SI]; **14.** $\int_0^{+\infty} \frac{1}{\sqrt{x^3 - x^4 - x^2}} dx$ [NO];

7. $\int_0^1 \frac{2}{\sqrt{e^x - 1}} dx$ [SI]; **15.** $\int_{-\infty}^1 x^4 e^x dx$ [SI];

8. $\int_0^1 \frac{1}{\ln x} dx$ [NO]; **16.** $\int_0^1 \frac{\sin x}{\sqrt{x}} dx$ [SI].

Stabilire se esistono valori di $\alpha \in \mathbb{R}$ per i quali i seguenti integrali sono convergenti:

$$17. \int_0^1 \frac{1 - \cos x}{x^\alpha} dx \quad [\alpha < 3];$$

$$21. \int_1^2 \frac{1}{(2-x)^\alpha} dx \quad [\alpha < 1];$$

$$18. \int_0^{+\infty} \frac{x}{(1+x^2)^\alpha} dx \quad [\alpha > 1];$$

$$22. \int_0^{+\infty} \frac{|\sin x|^\alpha}{x^2} dx \quad [\alpha > 1];$$

$$19. \int_3^{+\infty} \frac{1}{|3-x|^\alpha} dx \quad [\text{MAI}];$$

$$23. \int_0^{+\infty} x^{2\alpha} e^{-x} dx \quad [\alpha \geq 0];$$

$$20. \int_0^1 \frac{1}{(1+3x)^\alpha - (1+2x)^\alpha} dx \quad [\text{MAI}];$$

$$24. \int_{-2}^2 \frac{1}{(x^2-2)^\alpha} dx \quad [\alpha < 1].$$