An iterative algorithm for L1-TV constrained regularization in image restoration.
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Abstract

We consider the problem of restoring blurred images affected by impulsive noise. The adopted method restores the images by solving a sequence of constrained minimization problems where the data fidelity function is the \(l_1\) norm of the residual and the constraint, chosen as the image Total Variation, is automatically adapted to improve the quality of the restored images. Although this approach is general, we report here the case of vectorial images where the blurring model involves contributions from the different image channels (cross channel blur). A computationally convenient extension of the Total Variation function to vectorial images is used and the results reported show that this approach is efficient for recovering nearly optimal images.

Multichannel L1-TV Image Restoration

\[ HU = Y + \xi = Y^b \]

- Blurring kernel \(H\) block matrix \((H_{n,k}), (n,k)\) 1,\ldots,\(p\).
- \(H_{n,k}, n \neq k\): the cross-channel blur between \(n\)-th and \(k\)-channels.
- \(H_{n,n}\): within-channel blur.
- \(Y = \{Y_1, Y_2, \ldots, Y_p\}\): \(p\)-channels blurred image

\[ Y_o = \sum_{k=1}^{p} H_{n,k} = U_k, \quad n = 1, \ldots, p \quad Y_o \in R^{M \times N} \quad (2) \]

\[ Y^d\): noisy image corrupted by salt and pepper noise \(\xi^d\)

\[ \text{Ill-posed problem } \Rightarrow \text{Regularization method } \min_{U} \|HU - Y^d\|_1 \quad \text{s.t. } MTV(U) \leq \gamma \quad (3) \]

\[ MTV(U) \text{ discrete Multichannel Total Variation: } \]

\[ MTV(U) = \sum_{j=1}^{M} \sum_{i=1}^{N} (|\nabla U|_{i,j}^2 + |\nabla U|_{i,j} + \cdots + |\nabla U|_{i,j}^2)^{1/2}. \]

Since \(\gamma\) is UNKNOWN, we solve a sequence of problems (3) with \(\gamma = \gamma_j, j = 1, \ldots, r\), where \(\gamma_j\) is updated taking into account the \(l_1\) residual values.

Algorithm

- For each \(\gamma_j\) problem (3) is solved in Lagrangian dual form:

\[ \max_{\lambda} \min_{U} \{\|HU - Y^d\|_1 + \lambda (MTV(U) - \gamma_j)\} \]

1. find \(\lambda\) s.t. \(MTV(U(\lambda)) - \gamma_j = 0\) (Bisection+secant method) \([1]\)
2. \(U(\lambda) = \arg\min_{U}\{\|HU - Y^d\|_1 + \lambda (MTV(U) - \gamma_j)\}\) (FTVD \([2]\))

- Compute \(\gamma_0 = MTV(U(F))\) where \(U(F)\) is a low pass filtered starting image obtained by solving:

\[ U(F) = \arg\min_{U}\{\|HU - Y^d\|_1 + \lambda F MTV(U)\} \]

- \(\lambda F\) is computed by undersampling the the spectra of the blurring matrices \(H_{n,k}\) and taking the minimum of the Fourier coefficients.

- CL1TV Algorithm for iteratively updating of the smoothing values \(\gamma_j\):

\[ j = 0; \]

\[ \text{repeat} \]

\[ j = j + 1, \quad \gamma_j = \gamma_{j-1}(1 + P); \]

\[ \text{compute } U(\lambda) \text{ by solving (3)}; \]

\[ \gamma_j = \frac{\|HU(\lambda) - Y^d\|_1}{\|H - U(\lambda)\|_1}; \]

\[ D_j = (\gamma_j - \gamma_{j-1})/\gamma_j \quad \text{until } D_j < -P_{tol} \]

- \(P\) relative change in the values of \(\gamma_j\) \((P = 0.1); \quad P_{tol} = 1.e - 4\)

Numerical results

<table>
<thead>
<tr>
<th>Test (nl)</th>
<th>(E_0)</th>
<th>(E_r)</th>
<th>(it)</th>
<th>(it_{FTVD})</th>
<th>(it_{secant})</th>
</tr>
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<tbody>
<tr>
<td>I1 40%</td>
<td>0.477</td>
<td>0.139</td>
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<td>33</td>
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</tr>
</tbody>
</table>

Table: CL1TV algorithm results. \(E_0\) is the relative error of the recorded image, \(E_r\) the relative error of the restored image, \(it\), the number of elements of the sequence \(\gamma_j\), \(it_{FTVD}\) the total number of iterations of the FTVD method.

Figure: Relative errors and L1 residuals for the tests with medium noise

Figure: \(l_1\) image

Figure: \(l_2\) image

References
