An iterative algorithm for L1-TV constrained regularization in image restoration.

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Abstract

We consider the problem of restoring blurred images affected by impulsive noise. The adopted method restores the images by solving a sequence of constrained minimization problems where the data fidelity function is the ℓ_1 norm of the residual and the constraint, chosen as the image Total Variation, is automatically adapted to improve the quality of the restored images. Although this approach is general, we report here the case of vectorial images where the blurring model involves contributions from the different image channels (cross channel blur). A computationally convenient extension of the Total Variation function to vectorial images is used and the results reported show that this approach is efficient for recovering nearly optimal images.

Multichannel L1-TV Image Restoration

$$\boxed{\mathcal{H}U = Y + \zeta^\delta = Y^\delta}, \quad U = [U_1, Y_2, \dots, U_p] \text{ unknown image} \tag{1}$$

- Blurring kernel $\mathcal{H}$ block matrix $\{H_{n,k}\}$, (n,k) $1,\dots,p$.
  - $H_{n,k}$, $n \neq k$: the cross-channel blur between n-th and k-channels.
  - $H_{k,k}$: within-channel blur.
- $Y = [Y_1, Y_2, \dots, Y_p]$: p-channels blurred image

$$Y_n = \sum_{k=1}^{p} H_{n,k} * U_k, \quad n = 1,\dots,p \quad Y_n \in \mathcal{R}^{M\times N} \tag{2}$$

- Y^δ: noisy image corrupted by salt and pepper noise ζ^δ

Ill-posed problem $\Rightarrow$ Regularization method

$$\boxed{min_U \|\mathcal{H}U - Y^\delta\|_1 \ \ s.t. \ \ MTV(U) \le \gamma} \tag{3}$$

- $MTV(U)$ discrete Multichannel Total Variation:

$$MTV(U) = \sum_{j=1}^{M}\sum_{i=1}^{N} \left(|\nabla U|^2_{i,j,1} + |\nabla U|^2_{i,j,2} + \cdots + |\nabla U|^2_{i,j,p}\right)^{1/2}.$$

- Since γ is UNKNOWN, we solve a sequence of problems (3) with $\gamma = \gamma_j$, $j = 1,\dots,r$, where γ_j is updated taking into account the $l1$ residual values.

Algorithm

- For each γ_j problem (3) is solved in Lagrangian dual form :

$$\max_{\lambda}\left\{\min_{U}\left\{\|\mathcal{H}U - Y^\delta\|_1 + \lambda\left(MTV(U) - \gamma_j\right)\right\}\right\} \tag{4}$$

1. find $\hat\lambda$ s.t. $MTV(U(\hat\lambda)) - \gamma_j = 0$ (Bisection+secant method) [1]
2. $U(\hat\lambda) = argmin_U \|\mathcal{H}U - Y^\delta\|_1 + \hat\lambda\left(MTV(U) - \gamma_j\right)$ (FTVD [2])

- Compute $\gamma_0 = MTV(U^{(F)})$ where $U^{(F)}$ is a low pass filtered starting image obtained by solving:

$$U^{(F)} = argmin_U \|\mathcal{H}U - Y^\delta\|_1 + \lambda_F MTV(U)$$

- λ_F is computed by undersampling the the spectra of the blurring matrices $H_{n,k}$ and taking the minimum of the Fourier coefficients.
- CL1TV Algorithm for iteratively updating of the smoothing values γ_j:

```
j = 0;
repeat
  j = j + 1,  γ_j = γ_{j-1}(1 + P);
  compute U^(j) by solving (3);
  r_j = ||HU^(j) − Y^δ||_1;
  D_j = (r_j − r_{j-1})/(γ_j − γ_{j-1})
until D_j < −P_tol
```

- P relative change in the values of γ_j ($P = 0.1$); $P_{tol} = 1.e-4$

Numerical results

$$\mathcal{H} = \begin{pmatrix} .7G(21,11) & .15G(21,11) & .15G(21,11) \\ .1G(21,11) & .8G(21,11) & .1G(21,11) \\ .2G(21,11) & .2G(21,11) & .6G(21,11) \end{pmatrix}$$

where $G(h,\sigma)$ is a Gaussian function of size $h = 21$ pixels and variance $\sigma = 11$ pixels.

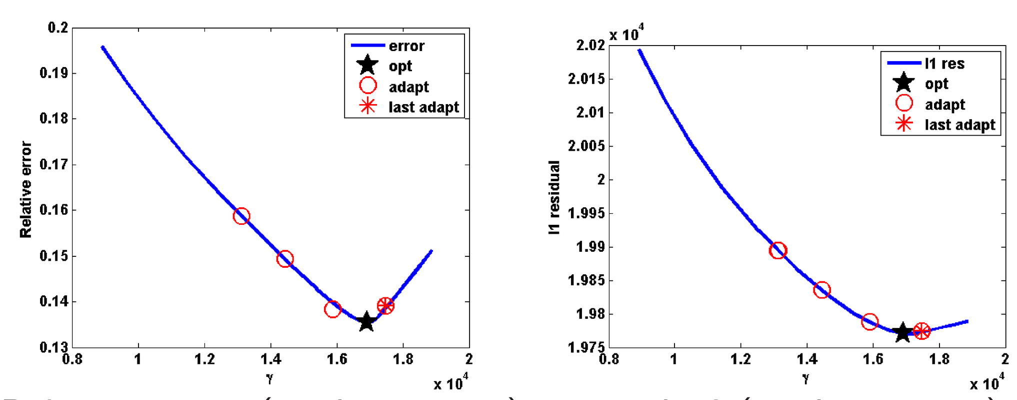
Test setting:

- Salt & Pepper noise (`imnoise` matlab function)
- two levels of noise: 40% and 80% of corrupted pixels
- maximum number of CL1TV iterations is 5

Test (nl)	$E0$	Er	it_γ	it_λ	it_{FTVD}
I1 40%	0.477	0.139	5	33	15768
I1 80%	0.600	0.145	5	24	10941
I2 40%	0.306	0.050	5	37	5670
I2 80%	0.4166	0.051	5	27	4021

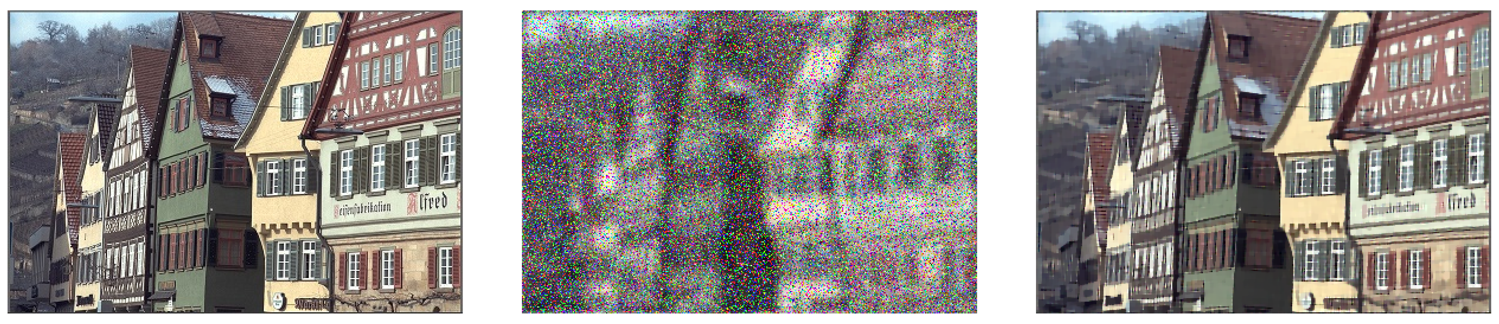
Table: CL1TV algorithm results. $E0$ is the relative error of the recorded image, Er the relative error of the restored image, it_γ the number of elements of the sequence γ_i, it_λ the total number of iterations of the bisection and secant methods, it_{FTVD} the total number of iterations of the FTVD method.

Figure: Relative errors and L1 residuals for the tests with medium noise



Relative errors (medium noise) residuals(medium noise)

Figure: I1 image



Original image Blurred noisy image Restored image

Figure: I2 image.

Original image Blurred noisy image Restored image

References

[1] Chen K, Piccolomini E and Zama F 2013 *Numerical Algorithms*

[2] Yang J, Zhang Y and Yin W 2009 *Siam J. Sci. Comput.* **31** 2482–2865