



$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right)^s f(x_0, y_0)$$

$$\frac{\partial^s f(x_0, y_0)}{\partial x^s} + \binom{s}{2} \frac{\partial^s f(x_0, y_0)}{\partial x^{s-1} \partial y} + \dots + \frac{\partial^s f(x_0, y_0)}{\partial y^s}$$

$$\begin{aligned}
 f(x, y) &= \cancel{f(x_0, y_0)} = 0 \\
 &+ \left((x-x_0) \frac{\partial}{\partial x} + (y-y_0) \frac{\partial}{\partial y} \right) f(x_0, y_0) \\
 &+ \dots \\
 &+ \frac{1}{(s-1)!} \left((x-x_0) \frac{\partial}{\partial x} + (y-y_0) \frac{\partial}{\partial y} \right)^{s-1} f(x_0, y_0) \\
 &+ \frac{1}{s!} \left((x-x_0) \frac{\partial}{\partial x} + (y-y_0) \frac{\partial}{\partial y} \right)^s f(x_0, y_0) \\
 &+ \dots
 \end{aligned}$$

$$\left. \begin{aligned}
 &f(x, y) = 0 \\
 &(y-y_0) = k(x-x_0)
 \end{aligned} \right\}$$

$\alpha \quad \rightarrow$

$$\left. \begin{aligned}
 &(y-y_0) = k(x-x_0) \\
 &0 = (x-x_0) \left(\frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right) f(x_0, y_0) + \\
 &+ \dots \\
 &+ \frac{1}{(s-1)!} (x-x_0)^{s-1} \left(\frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^{s-1} f(x_0, y_0) + \\
 &+ \frac{1}{s!} (x-x_0)^s \left(\frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^s f(x_0, y_0) + \\
 &+ \dots
 \end{aligned} \right\}$$

k coeff. ang di una
tang. nel punto

$\Leftrightarrow k$ soddisfa

$$\left(\frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right) f(x_0, y_0) = 0$$

$$(y - y_0) = k(x - x_0)$$

$$\alpha(y - y_0) = \beta(x - x_0)$$

$$\left(\alpha \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial y} \right) f(x_0, y_0) = 0$$

$$\square \text{ 50 } \Delta = z$$

$$\alpha \frac{\partial^2 f(x, y, z)}{\partial x^2} + 2\beta \frac{\partial^2 f(x, y, z)}{\partial x \partial y} + \gamma \frac{\partial^2 f(x, y, z)}{\partial y^2} = 0$$

$$\alpha \left(\frac{\partial^2}{\partial x^2} + 2\beta \frac{\partial^2}{\partial y^2} \right) = 0$$

22/7/'09 es 1a

$$C: x^4 - y^4 + xy = 0$$

trovare i punti multipli e tangenti

$$f_x = 4x^3 + y$$

$$f_{xx} = 12x^2$$

$$f_y = -4y^3 + x$$

$$f_{xy} = 1$$

$$f_{yy} = -12y^2$$

$$\begin{cases} f=0 \\ f_x=0 \\ f_y=0 \end{cases} \begin{cases} 256y^8 - y^4 + 4y^4 = 0 \\ f_x = 0 \\ x = 4y^3 \end{cases}$$

$$y^4 (256y^8 + 3) = 0$$

$$\begin{cases} y = 0 \\ x = 0 \\ x = 0 \end{cases} (0,0)$$

$$0 + 1k + 0k^2 = 0 \quad / \quad k = 0$$

$$0\alpha^2 + \alpha\beta + 0\beta^2 = 0 \quad / \quad k = \infty$$

$$\alpha\beta = 0$$

$$\alpha x = \beta y$$

Asintoti di C:

$$x^4 - y^4 + xy = 0$$

$$X_1^4 - X_2^4 + X_1 X_2 X_0^2 = 0$$

$$F(X_0, X_1, X_2) = \begin{array}{cc} P_{\infty}^I & P_{\infty}^{II} \\ 0 & 0 \end{array}$$

$$F_0 = 2X_0 X_1 X_2 \quad \begin{array}{cc} 0 & 0 \\ 4 & 4 \end{array}$$

$$F_1 = 4X_1^3 + X_2 X_0^2 \quad \begin{array}{cc} 4 & 4 \\ -4 & 4 \end{array}$$

$$F_2 = -4X_2^3 + X_1 X_0^2 \quad \begin{array}{cc} -4 & 4 \\ 4 & 4 \end{array}$$

$$\begin{cases} X_1^4 - X_2^4 + \cancel{X_1 X_2 X_0^2} = 0 \\ X_0 = 0 \end{cases}$$

$$\begin{cases} X_1^4 - X_2^4 = 0 \quad (X_1^2 + X_2^2)(X_1^2 - X_2^2) = 0 \\ X_0 = 0 \end{cases}$$

$$\begin{cases} X_1 = \pm X_2 & P_{\infty}^I = (0, 1, 1) \\ X_0 = 0 & P_{\infty}^{II} = (0, 1, -1) \end{cases}$$

tang. in P_{∞}^I :

$$0 X_0 + 4X_1 - 4X_2 = 0$$

$$4x - 4y = 0$$

tang. in P_{∞}^{II} :

$$0 X_0 + 4X_1 + 4X_2 = 0$$

$$4x + 4y = 0$$