

ES. 14 Σ :

$$x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2 + 2y^2z^2 - 14x^2 - 14y^2 - 10z^2 + 25 = 0$$

$$F(x, y, z) =$$

$$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \\ F_z = 0 \end{cases}$$

$$F_x = 4xz^2 + 4xy^2 + 4x^3 - 28x = 4x(x^2 + y^2 + z^2 - 7)$$

$$F_y = 4yz^2 + 4y^3 + 4x^2y - 28y = 4y(x^2 + y^2 + z^2 - 7)$$

$$F_z = 4z^3 + 4y^2z + 4x^2z - 20z = 4z(x^2 + y^2 + z^2 - 5)$$

$$\begin{cases} F = 0 \\ x(x^2 + y^2 + z^2 - 7) = 0 \\ y(x^2 + y^2 + z^2 - 7) = 0 \\ z(x^2 + y^2 + z^2 - 5) = 0 \end{cases}$$

$$\textcircled{1} \begin{cases} F \neq 0 \\ x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

$$\begin{array}{l}
 \textcircled{2} \\
 F = 0 \\
 (x^2 + y^2 + z^2 - 7) = 0 \\
 y = 0 \\
 z = 0
 \end{array}
 \left.
 \begin{array}{l}
 F \neq 0 \\
 x = \pm \sqrt{7} \\
 y = 0 \\
 z = 0
 \end{array}
 \right\}$$

$$\begin{array}{l}
 \textcircled{3} \\
 F \neq 0 \\
 x = 0 \\
 (x^2 + y^2 + z^2 - 7) = 0 \\
 z = 0
 \end{array}$$

$$\begin{array}{l}
 \textcircled{4} \\
 F = 0 \\
 x = 0 \\
 y = 0 \\
 x^2 + y^2 + z^2 - 5 = 0
 \end{array}
 \left.
 \begin{array}{l}
 F = 0 \\
 x = 0 \\
 y = 0 \\
 z = \pm \sqrt{5}
 \end{array}
 \right\}$$

$$\begin{array}{l}
 \textcircled{5} \\
 F = 0 \\
 x^2 + y^2 + z^2 - 7 = 0 \\
 \text{idem} \\
 z = 0
 \end{array}
 \left.
 \begin{array}{l}
 F \neq 0 \\
 x^2 + y^2 = 7 \\
 z = 0
 \end{array}
 \right\}$$

$$\begin{aligned}
 (x^2 + y^2)^2 - 14(x^2 + y^2) + 25 &= \\
 = 49 - 98 + 25 &\neq 0
 \end{aligned}$$

$$\begin{array}{l} \textcircled{2} \quad F = 0 \\ x = 0 \\ x^2 + y^2 + z^2 - 7 = 0 \\ \dots - 5 = 0 \end{array} \quad \begin{array}{l} \textcircled{7} \quad F = 0 \\ x^2 + y^2 + z^2 - 7 = 0 \\ y = 0 \\ \dots - 5 = 0 \end{array}$$

$$\textcircled{8} \quad \begin{array}{l} F = 0 \\ \dots - 7 = 0 \\ \dots - 7 = 0 \\ \dots - 5 = 0 \end{array}$$

Punti multi $M(0, 0, \sqrt{5})$

$$F_{xx} = 4z^2 + 4y^2 + 12x^2 - 28 = -8$$

$$F_{xy} = 8xz = 0$$

$$F_{xz} = 8xz = 0$$

$$F_{yy} = 4z^2 + 12y^2 + 4x^2 - 28 = -8$$

$$F_{yz} = 8yz = 0$$

$$F_{zz} = 12z^2 + 4y^2 + 4x^2 - 20 = 40$$

$$-8(x-0)^2 - 8(y-0)^2 + 40(z-\sqrt{5})^2 = 0$$