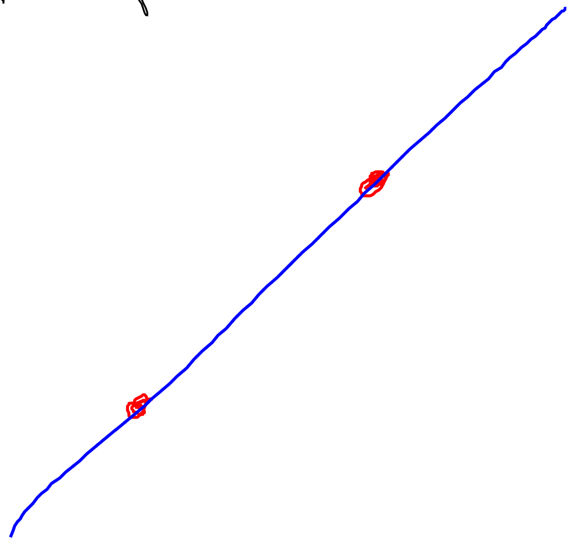
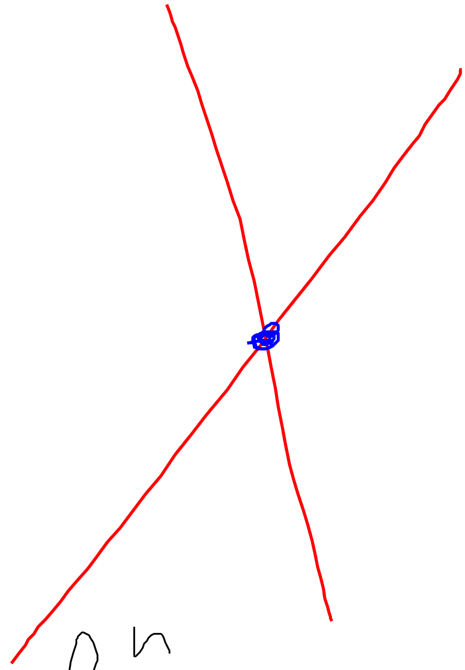
Nel piano





iperpiano π di $\mathbb{C}^n$

$$a_1 x_1 + \dots + a_n x_n + b = 0$$

$$x_1 = \frac{X_1}{X_0}, \dots, x_n = \frac{X_n}{X_0} \qquad X_0 \neq 0$$

$$a_1 \frac{X_1}{X_0} + \dots + a_n \frac{X_n}{X_0} + b = 0 \quad \cdot X_0$$

$$a_1 X_1 + \dots + a_n X_n + b X_0 = 0$$

iperpiano di $\mathbb{P}^n$ è l'ampliamento proiettivo di π.

$r: \quad bx + cy + a = 0$

$s: \quad ex + fy + d = 0$

$$x = \frac{X_1}{X_0} \qquad y = \frac{X_2}{X_0}$$

$$\begin{cases} aX_0 + bX_1 + cX_2 = 0 \\ dX_0 + eX_1 + fX_2 = 0 \end{cases}$$

$$(X_0, X_1, X_2) \sim \left(\begin{vmatrix} b & c \\ e & f \end{vmatrix}, -\begin{vmatrix} a & c \\ d & f \end{vmatrix}, \begin{vmatrix} a & b \\ d & e \end{vmatrix} \right)$$

$$\begin{cases} r: \; x + y + 2 = 0 \\ s: \; x - y + 4 = 0 \end{cases}$$

$$\begin{cases} x + y = -2 \\ \quad -2y = -2 \end{cases} \qquad \begin{cases} x = -3 \\ y = 1 \end{cases}$$

$$\begin{cases} 2X_0 + X_1 + X_2 = 0 \\ 4X_0 + X_1 - X_2 = 0 \end{cases}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 4 & 1 & -1 \end{pmatrix}$$

$$(X_0, X_1, X_2) \sim \left(\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} \right) =$$

$$= (-2, 6, -2)$$

$$P = r \cap s = \left(\frac{6}{-2}, \frac{-2}{-2} \right) = (-3, 1)$$

$r: \; x + y + 2 = 0 \quad (l, m) \sim (1, -1)$

$s: \; 4x + 4y + 6 = 0 \quad (l, m) \sim (4, -4)$

$$\underbrace{\begin{pmatrix} 1 & 1 & 2 \\ 4 & 4 & 6 \end{pmatrix}}_{A \quad C}$$

$r A = 1 \neq r C = 2$

$$\begin{cases} 2X_0 + X_1 + X_2 = 0 \\ 6X_0 + 4X_1 + 4X_2 = 0 \end{cases} \qquad \begin{pmatrix} 2 & 1 & 1 \\ 6 & 4 & 4 \end{pmatrix}$$

$$(X_0, X_1, X_2) \sim \left(\begin{vmatrix} 1 & 1 \\ 4 & 4 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 6 & 4 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 6 & 4 \end{vmatrix} \right) =$$

$$= (0, -2, 2) \sim (0, 1, -1)$$

$l \; m$

PROBL

$\mathbb{A}^3$ $\quad r: \begin{cases} 2x+y-z-2=0 \\ x-3y+z+1=0 \end{cases}$

$P=(1,2,3)=(1,1,2,3)$

Trovare il piano Π per r e P.

Nell'ampl. pro. $\mathbb{P}^3$

(ampli pro. di) r : $\begin{cases} -2X_0+2X_1+X_2-X_3=0 \\ X_0+X_1-3X_2+X_3=0 \end{cases}$

Generico piano per r:

$\alpha(-2X_0+2X_1+X_2-X_3)+\beta(X_0+X_1-3X_2+X_3)=0$

impongo il pass. per $(1,1,2,3)$

$\alpha(-2+2+2-3)+\beta(1+1-6+3)=0$

$-\alpha-\beta=0 \quad \beta=-\alpha$

scelgo $(\alpha,\beta)=(1,-1)$

$-3X_0+X_1+4X_2-2X_3=0$

$$\boxed{x+4y-2z-3=0}$$

A^3 $\quad r: \begin{cases} 2x+y-z-2=0 \\ x-3y+z+1=0 \end{cases}$

$s: \begin{cases} x=3t-1 \\ y=-5t \\ z=2t+4 \end{cases}$ $\quad (l,m,n) \sim (3,-5,2)$

Trovare il piano π' passante per r e $\parallel s$

$$\alpha(-2X_0+2X_1+X_2-X_3)+\beta(X_0+X_1-3X_2+X_3)=0$$

Impongo il passaggio per il punto improprio (dell'ampliamento proiettivo) della retta s:

$$S_\infty \equiv (0,3,-5,2)$$

$$\alpha(0+6-5-2)+\beta(0+3+15+2)=0$$

$$-\alpha+20\beta=0$$

$$\alpha=20\beta \quad \text{scelgo } (\alpha,\beta)=(20,1)$$

$$-39X_0+41X_1+17X_2-19X_3=0$$

$$41x+17y-19z-39=0$$