11/9/'09 Es. 4 a

Trovare i punti multipli reali e le tangenti in uno di essi per

$$\mathcal{C}:\ \underbrace{x^2y - xy^2 - x + y}_{F(x,y)=} = 0$$

$$F_x = 2xy - y^2 - 1$$

$$F_y = x^2 - 2xy + 1$$

Cerco eventuali soluzioni di

$$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \end{cases} \qquad \begin{cases} F = 0 \\ 2xy - y^2 - 1 = 0 \\ x^2 - 2xy + 1 = 0 \end{cases}$$

$$\begin{cases} F = 0 \\ 2xy = y^2 + 1 \\ 2xy = x^2 + 1 \end{cases} \qquad \begin{cases} F = 0 \\ 2xy = y^2 + 1 \\ x^2 \cancel{+1} = y^2 \cancel{+1} \end{cases}$$

$$\begin{cases} F = 0 \\ 2xy = y^2+1 \\ x = \pm y \end{cases}$$

① $$\begin{cases} F = 0 \\ 2xy = y^2+1 \\ x = y \end{cases}$$

② $$\begin{cases} F = 0 \\ 2xy = y^2+1 \\ x = -y \end{cases}$$

① $$\begin{cases} F = 0 \\ 2x^2 = x^2+1 \\ x = y \end{cases} \qquad \begin{cases} x^2y - xy^2 - x + y = 0 \\ x^2 = 1 \\ x = y \end{cases} \qquad \begin{cases} F = 0 \\ x = \pm 1 \\ x = y \end{cases}$$

1′, 1″

1′ $$\begin{cases} F = 0 \\ x = 1 \\ x = y \end{cases} \qquad \begin{cases} 1 - 1 - 1 + 1 = 0 \quad \checkmark \\ x = y = 1 \end{cases} \qquad M_1 = (1,1)$$

1″ $$\begin{cases} F = 0 \\ x = -1 \\ x = y \end{cases} \qquad \begin{cases} -1 + 1 + 1 - 1 = 0 \quad \checkmark \\ x = y = -1 \end{cases} \qquad M_2 = (-1,-1)$$

② $$\begin{cases} F = 0 \\ 2xy = y^2+1 \\ x = -y \end{cases} \qquad \begin{cases} F = 0 \\ -2x^2 = x^2+1 \\ x = -y \end{cases} \qquad \cancel{\begin{cases} F = 0 \\ 3x^2 = -1 \\ x = -y \end{cases}}$$

$$F_x = 2xy - y^2 - 1$$

$$F_y = x^2 - 2xy + 1$$

In $M_1 \equiv (1,1)$

$F_{xx} = 2y$ $\quad 2$

$F_{xy} = 2x - 2y$ $\quad 0$

$F_{yy} = -2x$ $\quad -2$

Generica retta per M_1:

$$y - 1 = k(x-1)$$

$$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 F(1,1) = 0$$

$$2 + 0k - 2k^2 = 0 \qquad 2 - 2k^2 = 0$$
$$k = \pm 1$$

Tangenti in M_1:

$$y - 1 = x - 1$$
$$y - 1 = -x + 1$$

Alternativamente: posso scrivere l'eq. complessiva delle tangenti

$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)^2 F(x_0,y_0)=0$$

$$2(x-1)^2-2(y-1)^2=0$$

22/7/'09 Es 1 a

Punti multipli e tangenti in esso di

$$\mathcal{C}: \underbrace{x^4-y^4+xy=0}_{F(x,y)=}$$

$$F_x=4x^3+y$$

$$F_y=-4y^3+x$$

Cerco sol. di

$$\begin{cases} F=0 \\ F_x=0 \\ F_y=0 \end{cases}$$

$$\begin{cases} F=0 \\ 4x^3+y=0 \\ x=4y^3 \end{cases} \qquad \begin{cases} F=0 \\ 256y^9+y=0 \\ x=4y^3 \end{cases}$$

$$\begin{cases} F = 0 \\ y(256y^8+1) = 0 \\ x = 4y^3 \end{cases} \qquad \begin{cases} F = 0 \\ y = 0 \\ x = 4y^3 \end{cases} \qquad \begin{cases} 0 = 0 \\ y = 0 \\ x = 0 \end{cases}$$

$\nexists$ in $\mathbb{R}$

punto multiplo: $O \equiv (0,0)$

$xy = 0$ è il compl. delle tangenti in O, perciò

Tangenti: $x = 0$ e $y = 0$

	In O
$F_x = 4x^3 + y$	0
$F_y = -4y^3 + x$	0
$F_{xx} = 12x^2$	0
$F_{xy} = 1$	1
$F_{yy} = -12y^2$	0

$$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 F(0,0) = 0 \qquad y = kx$$

$$0 + 1k + 0k^2 = 0 \qquad k = 0$$

$$\text{"}k = \infty\text{"} \qquad y = 0$$

$$x = 0$$

$$\frac{y}{m} = \frac{x}{l}$$

$$ly = mx$$

$$\left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^2 F(0,0) = 0$$

$$0l^2 + 1lm + 0m^2 = 0 \qquad lm = 0$$

$$l = 0 \qquad m = 0$$

$$x = 0 \qquad y = 0$$

$C': x^4 - y^4 + x^2 = 0$

In O

$F_x = 4x^3 + 2x$ $\quad 0$

$F_y = -4y^3$ $\quad 0$

$F_{xx} = 12x^2 + 2$ $\quad 2$

$F_{xy} = 0$ $\quad 0$

$F_{yy} = -12y^2$ $\quad 0$

$2 + 0k + 0k^2 = 0 \quad 2 = 0$

$l\ y = mx$

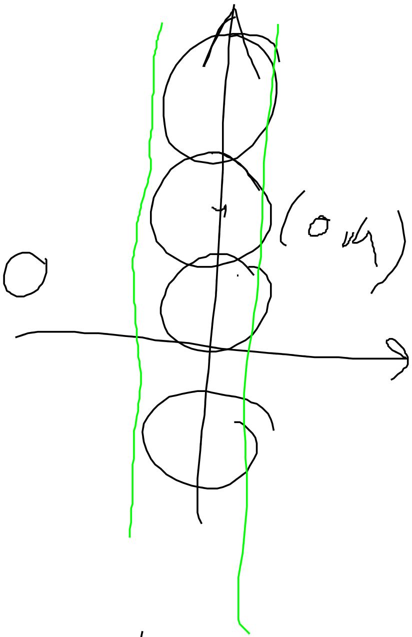
$2l^2 + 0lm + 0m^2 = 0 \quad 2l^2 = 0$

$x = 0$

2 volte

$C_u : (x-0)^2+(y-u)^2=1$

$$\underbrace{x^2+y^2+u^2-2uy-1}_{F(x,y;u)}=0$$



$F_u = 2u-2y$

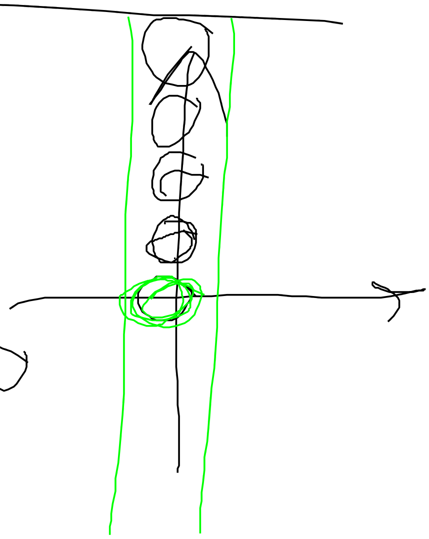
$$\begin{cases} x^2+y^2+u^2-2uy-1=0 \\ 2u-2y=0 \end{cases} \quad \begin{cases} \\ u=y \end{cases}$$

$$x^2+\cancel{y^2}+\cancel{y^2}-\cancel{2y\,y}-1=0$$

$$x^2-1=0$$

$C_u : (x-0)^2+(y-u^2)^2=1$

$$\underbrace{x^2+y^2+u^4-2yu^2-1}_{F(x,y;u)}=0$$



$$F_u = 4u^3 - 4yu \qquad \begin{cases} F = 0 \\ F_u = 0 \end{cases}$$

$$\begin{cases} x^2 + y^2 + u^4 - 2yu^2 - 1 = 0 \\ 4u(u^2 - y) = 0 \end{cases}$$

unione di ① $\begin{cases} \text{———} \\ u = 0 \end{cases}$

e di ② $\begin{cases} \text{———} \\ y = u^2 \end{cases}$

① $x^2 + y^2 - 1 = 0$

② $x^2 + \cancel{u^4} + \cancel{u^4} - 2\cancel{u^4} - 1 = 0 \quad x^2 - 1 = 0$