$\varkappa = \frac{1}{R}$ flessione

$\tau = \frac{1}{T}$ torsione

t versore tangente nel punto

n " sulla normale princ. " "

b " " binormale " "

$$\begin{pmatrix} t' \\ n' \\ b' \end{pmatrix} = \begin{pmatrix} 0 & \varkappa & 0 \\ -\varkappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix} \begin{pmatrix} t \\ n \\ b \end{pmatrix}$$

10/9/'10 Es. 45

$$\mathcal{L} : \begin{cases} x = t^2 - 1 \\ y = t^2 - t \\ z = -2t^2 \end{cases}$$

$A \leftrightarrow t = 0$
$= (-1, 0, 0)$

Triedro fondamentale in A

$x' = 2t \qquad y' = 2t - 1 \qquad z' = -4t$

$0 \qquad -1 \qquad 0 \qquad$ In A

$x'' = 2 \qquad y'' = 2 \qquad z'' = -4$

$2 \qquad 2 \qquad -4$

tangente $\quad \cancel{t : \dfrac{x+1}{0} = \dfrac{y-0}{-1} = \dfrac{z-0}{0}}$

$$\boxed{t : \begin{cases} x = -1 \\ y = -\alpha \\ z = 0 \end{cases} \qquad \begin{cases} x = -1 \\ z = 0 \end{cases}}$$

piano normale:

$$\pi_n : 0(x+1) - 1(y-0) + 0(z-0) = 0$$

$$\boxed{y = 0}$$

piano osculatore:

$$\begin{vmatrix} (x+1) & (y-0) & (z-0) \\ 0 & -1 & 0 \\ 2 & 2 & -4 \end{vmatrix} = 0$$

$$- \begin{vmatrix} (x+1) & z \\ 2 & -4 \end{vmatrix} = -\left(-4x-4-2z\right)=0$$

$$\boxed{\Pi_0 : 4x+2z+4=0 \quad (2x+z+2=0)}$$

normale principale:

$$\boxed{n :} \ \Pi_n \cap \Pi_0 \quad \boxed{\begin{cases} y=0 \\ 2x+z+2=0 \end{cases}}$$

binormale:

$$\boxed{b :} \ \frac{x+1}{2} = \cancel{\frac{y}{0}} = \frac{z}{1} \quad \boxed{\begin{cases} y=0 \\ x-2z+1=0 \end{cases}}$$

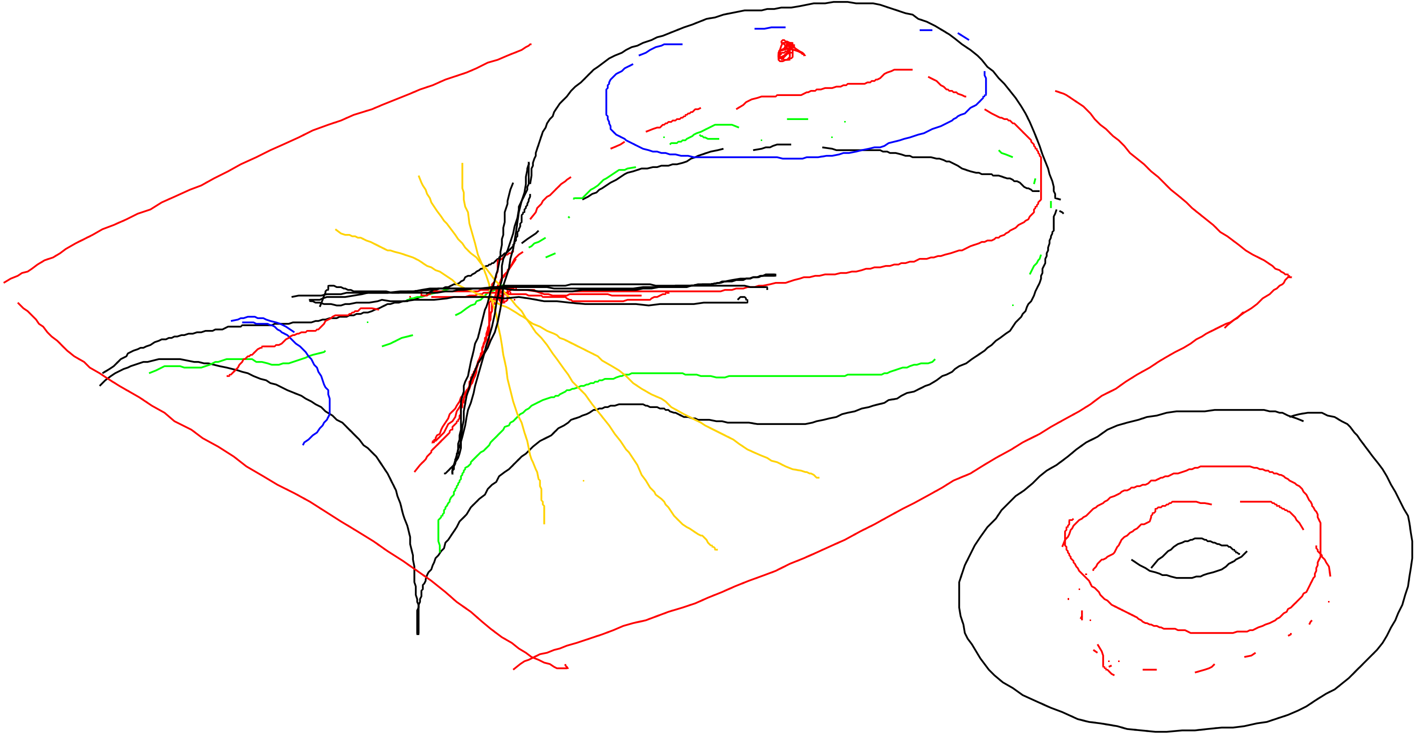
$$\begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad (l,m,n) \sim \left(\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, - \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} \right)$$

$$= (1,0,-2)$$

pian. rettificante:

$$\boxed{\Pi_r :} \ 1(x+1)+0(y-0)-2(z-0)=0$$

$$x-2z+1=0$$



20/7/'10 Es. 2b

S $F(x,y,z) =$ $x^2z + y^2z - 3x^2 - 3y^2 + z^2 - 6z + 9 = 0$

Trovare le tang. asint. in $P \equiv (\sqrt{3}, 0, 0)$

Generica retta per P:

$$\begin{cases} x = lu + \sqrt{3} \\ y = mu \\ z = nu \end{cases} \qquad \Phi(u) = F(x(u), y(u), z(u))$$

$\Phi(u) =$

$$= m^2nu^3 + l^2nu^3 + n^2u^2 + 2\sqrt{3}\,lnu^2 - 3m^2u^2 - 3l^2u^2 + $$
$$- 3nu - 6\sqrt{3}\,lu$$

$\Phi'(u) =$

$$= 3m^2nu^2 + 3l^2nu^2 + 2n^2u + 4\sqrt{3}\,lnu - 6m^2u + $$
$$- 6l^2u - 3n - 6\sqrt{3}$$

$$\Phi''(u) = 6m^2nu + 6\ell^2nu + 2n^2 + 4\sqrt{3}\,\ell n - 6m^2 - 6\ell^2$$

$$\Phi(0) = 0 \qquad \Phi'(0) = -3n - 6\sqrt{3}\,\ell$$

$$\Phi''(0) = 2n^2 + 4\sqrt{3}\,\ell n - 6m^2 - 6\ell^2$$

$$\begin{cases} \Phi'(0) = 0 \\ \Phi''(0) = 0 \end{cases} \qquad \begin{cases} -3n - 6\sqrt{3}\,\ell = 0 \qquad n + 2\sqrt{3}\,\ell = 0 \\ 2n^2 + 4\sqrt{3}\,\ell n - 6m^2 - 6\ell^2 = 0 \end{cases}$$

$$\begin{cases} n = -2\sqrt{3}\,\ell \\ \rule{4cm}{0.4pt} \end{cases}$$

$$\begin{cases} n = -2\sqrt{3}\,\ell \\ \cancel{24\ell^2} - \cancel{24\ell^2} - 6m^2 - 6\ell^2 = 0 \end{cases}$$

$$\begin{cases} n = -2\sqrt{3}\,\ell \\ m^2 = -\ell^2 \end{cases} \qquad \text{Scelgo } \ell = 1$$

Ottengo

$$\begin{cases} \ell = 1 \\ m = i \\ n = -2\sqrt{3} \end{cases} \qquad \begin{cases} \ell = 1 \\ m = -i \\ n = -2\sqrt{3} \end{cases}$$

$$t_1 : \left\{ \begin{array}{l} x = \sqrt{3} + u \\ y = i u \\ z = -2\sqrt{3}\, u \end{array} \right. \qquad t_2 \left\{ \begin{array}{l} x = \sqrt{3} + u \\ y = -i u \\ z = -2\sqrt{3}\, u \end{array} \right.$$

2017/10 Es 2a

§ trovare gli eventuali punti $F(x,y,z) = x^2z + y^2z + 3x^2 + 3y^2 + z^2 - 6z + 9 = 0$ in uno di essi

$$F_x = 2xz - 6x = 2x(z-3)$$

$$F_y = 2yz - 6y = 2y(z-3)$$

$$F_z = 2z + y^2 + x^2 - 6$$

$$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \\ F_z = 0 \end{cases} \qquad \begin{cases} F = 0 \\ 2x(z-3) = 0 \\ 2y(z-3) = 0 \\ 2z + y^2 + x^2 - 6 = 0 \end{cases}$$

1, 2, 3, 4

1 $$\begin{cases} F = 0 \\ x = 0 \\ y = 0 \end{cases} \qquad \begin{cases} F = 0 \\ x = 0 \\ y = 0 \\ 2z - 6 = 0 \end{cases} \qquad \begin{cases} 0 = 0 \\ x = 0 \\ y = 0 \\ z = 3 \end{cases}$$

$z = 3$

$M = (0, 0, 3)$

② $\begin{cases} F = 0 \\ z - 3 = 0 \\ y = 0 \\ 6 + 0 + x^2 - 6 = 0 \end{cases}$ $\quad \begin{cases} 0 = 0 \\ z = 3 \\ y = 0 \\ x = 0 \end{cases}$

③ $\begin{cases} F = 0 \\ x = 0 \\ z - 3 = 0 \\ 6 + 0 + y^2 - 6 = 0 \end{cases}$ $\quad \begin{cases} 0 = 0 \\ x = 0 \\ z = 3 \\ y = 0 \end{cases}$

④ $\begin{cases} F = 0 \\ z - 3 = 0 \\ z - 3 = 0 \\ 6 + x^2 + y^2 - 6 = 0 \end{cases}$ $\quad \begin{cases} F = 0 \\ z = 3 \\ z = 3 \\ x^2 + y^2 = 0 \end{cases}$

$S : x^2 z + y^2 z - 3x^2 - 3y^2 + z^2 - 6z + 9 = 0$

$\begin{cases} z(x^2 + y^2) - 3(x^2 + y^2) + z^2 - 6z + 9 = 0 \\ z = 3 \\ x^2 + y^2 = 0 \end{cases}$ $\quad \begin{cases} 3 \cdot 0 - 3 \cdot 0 + (3 - 3)^2 = 0 \\ z = 3 \\ x^2 + y^2 = 0 \end{cases}$

Ancora $M = (0, 0, 3)$

$$F_x = 2xz - 6x = 2x(z-3)$$
$$F_y = 2yz - 6y = 2y(z-3)$$
$$F_z = 2z + y^2 + x^2 - 6$$

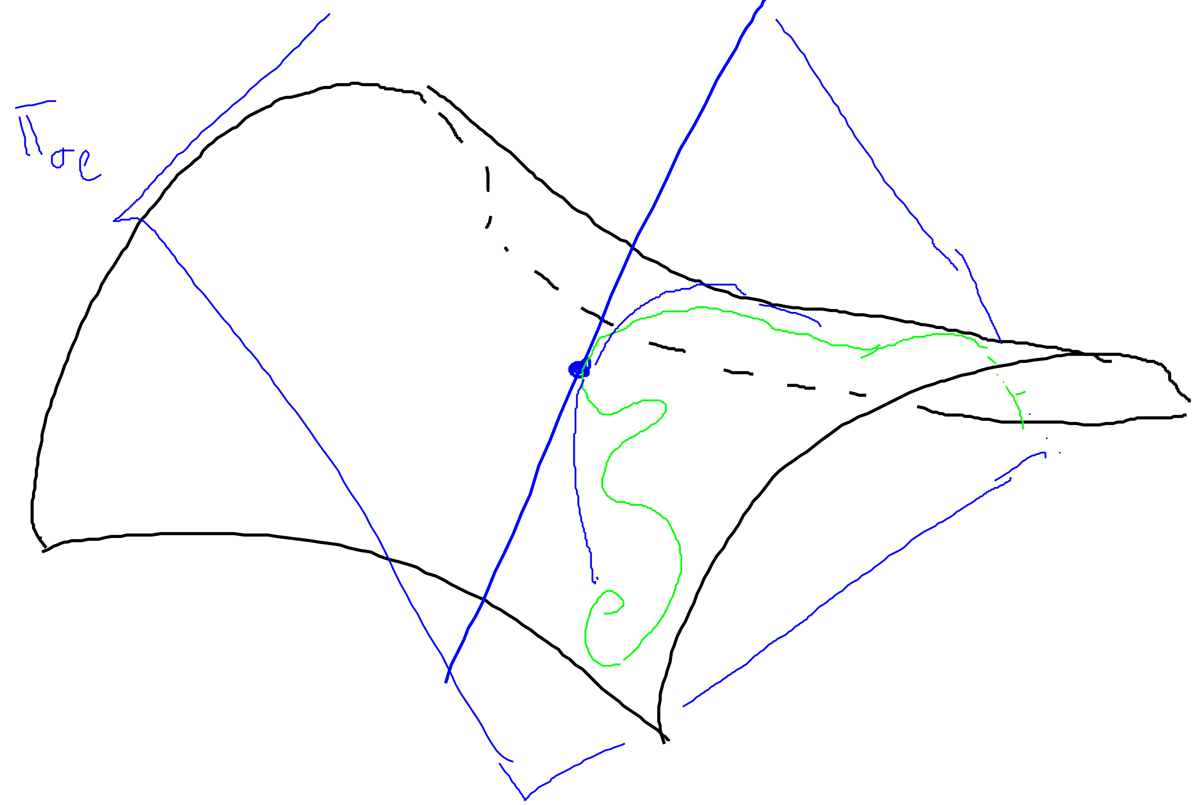
In $M = (0,0,3)$

$$F_{xx} = 2z - 6 \qquad 0$$
$$F_{xy} = 0 \qquad 0$$
$$F_{xz} = 2x \qquad 0$$
$$F_{yy} = 2z - 6 \qquad 0$$
$$F_{yz} = 2y \qquad 0$$
$$F_{zz} = 2 \qquad 2$$

Cono tangente in M:

$$\left[(x-0)\frac{\partial}{\partial x} + (y-0)\frac{\partial}{\partial y} + (z-3)\frac{\partial}{\partial z}\right]^2 F(0,0,3) = 0$$

$$\boxed{2(z-3)^2 = 0}$$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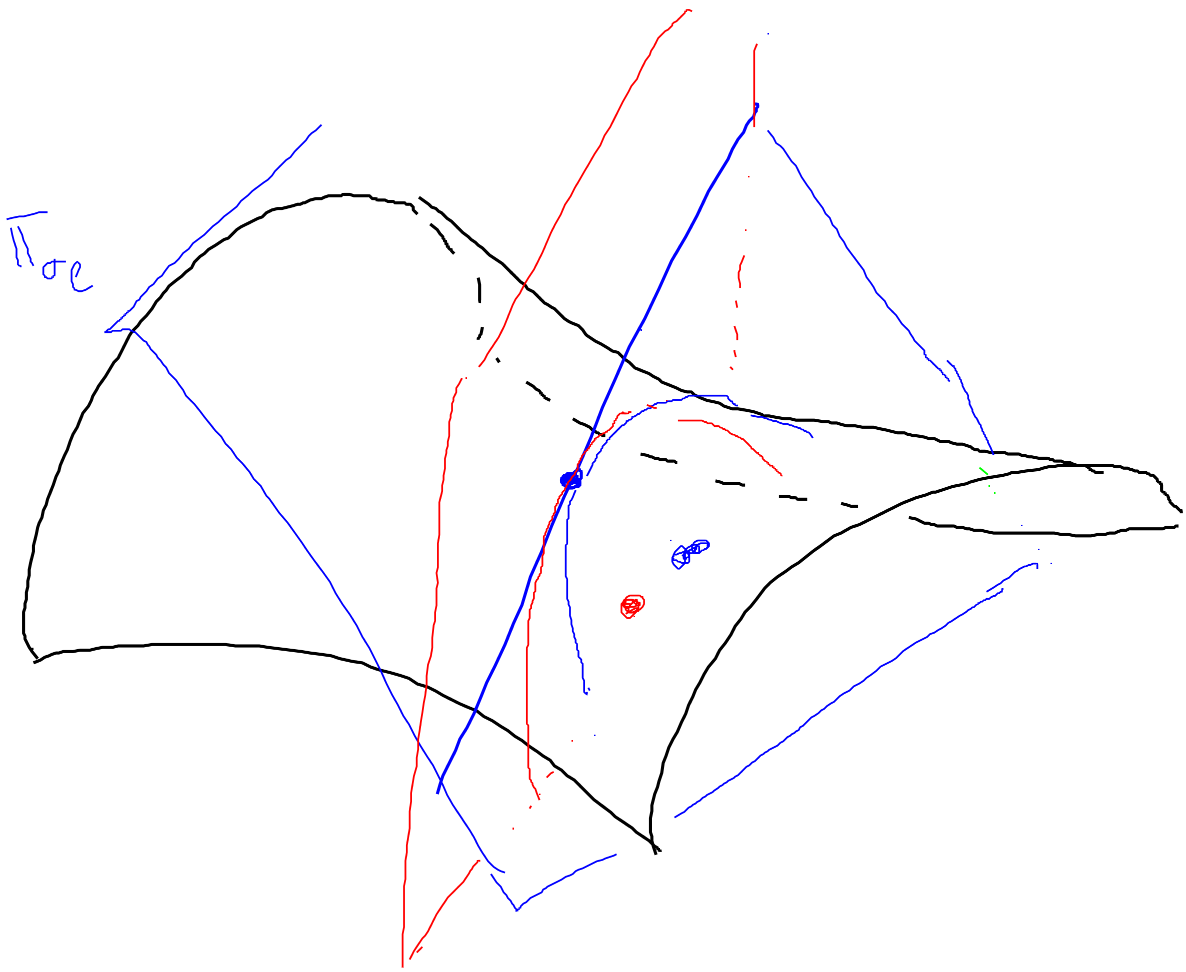
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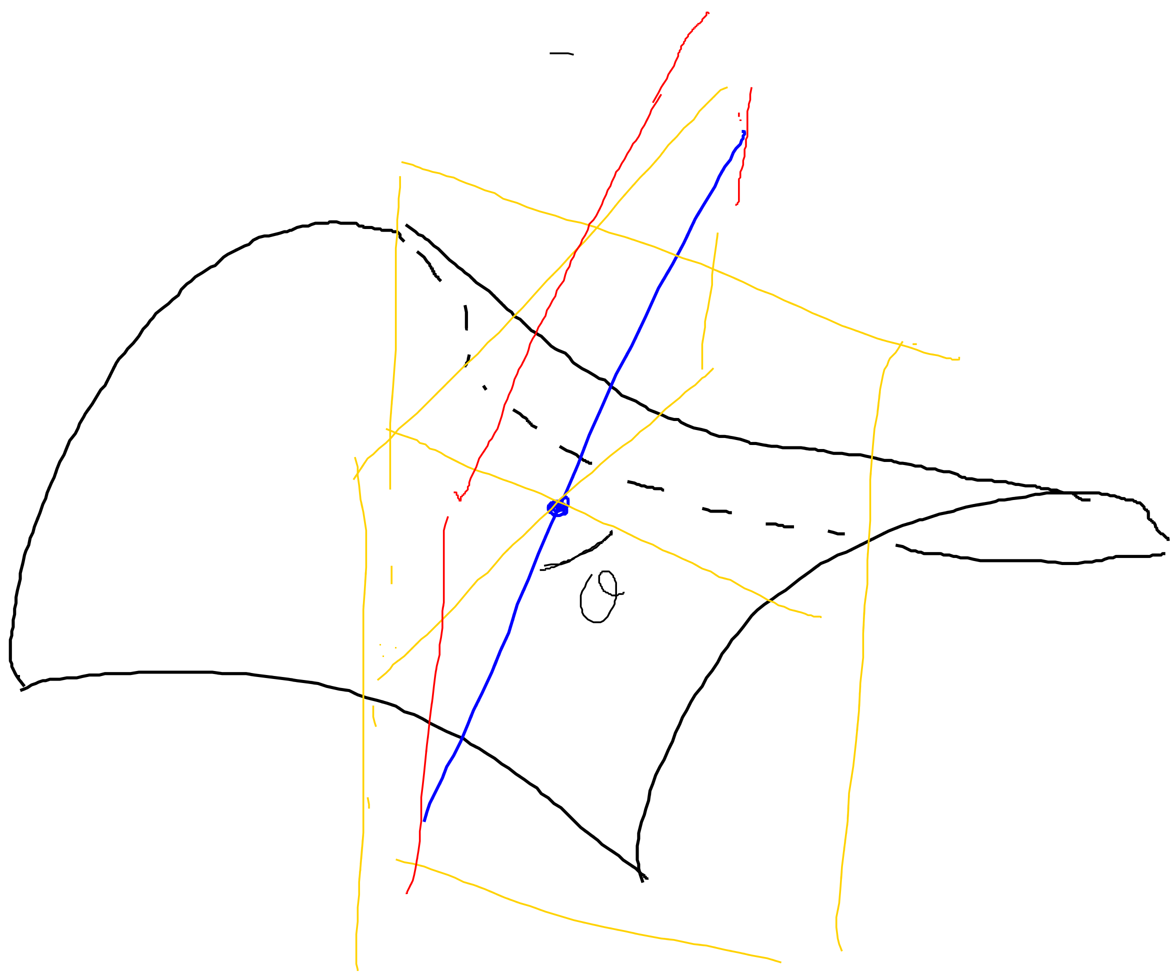
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