Una forma quadratica $q: V \to \mathbb{R}$

è detta definita positiva

negativa

se, $\forall v \in V - \{\bar{0}_V\}$ $q(v) > 0$

< 0

Viene detta indefinita

altrimenti

PROP — q. di matrice di Gram A risulta definita positiva negativa $\Longleftrightarrow$

$$\sigma(A) = (n,0)$$

$$(0,n)$$

COR q def pos neg. $\Longleftrightarrow$ il pol. car. di A ha termine noto $\neq 0$ e tutte variazioni permanenze

Andiamo a vedere com'è fatta l'immagine del cono reale

$$x_0^2 + x_1^2 - x_2^2 + 0x_3^2 = 0$$

Vertice $V \equiv (0,0,0,1)$

$P \in \mathrm{Im}$ $\quad P \equiv (a, b, \pm\sqrt{a^2+b^2}, c)$

Retta PV :

$$\lambda(a, b, \pm\sqrt{a^2+b^2}, c) + \mu(0, 0, 0, 1) =$$

$$= (\lambda a, \lambda b, \pm\lambda\sqrt{a^2+b^2}, \lambda c + \mu)$$

$\in Im$

$$(\lambda a)^2 + (\lambda b)^2 - (\lambda\sqrt{a^2+b^2})^2 + 0(\lambda c+\mu)^2 =$$

$$= \lambda^2 a^2 + \lambda^2 b^2 - \lambda^2 (a^2 + b^2) = 0$$

Vediamo com'è fatta l'Im di $X_0^2+X_1^2-X_2^2-X_3^2=0$ $\begin{pmatrix}1 & & & 0\\ & 1 & & \\ & & -1 & \\ 0 & & & -1\end{pmatrix}$

$P\equiv\left(a,b,c,\pm\sqrt{a^2+b^2-c^2}\right)$ a,b,c costanti

$\in Im$ $\quad a^2+b^2-c^2 \geq 0$

$\tau(P): aX_0+bX_1-cX_2\mp\sqrt{}\,X_3=0$

$$\tau(P): \begin{cases} x_0 = \alpha \\ x_1 = \beta \\ x_2 = \gamma \\ x_3 = \pm \dfrac{a\alpha + b\beta - c\gamma}{\sqrt{}} \end{cases}$$

$$\alpha^2 + \beta^2 - \gamma^2 - \frac{(a\alpha + b\beta - c\gamma)^2}{a^2 + b^2 - c^2} =$$

è la restrizione a $\tau(P)$ della forma quadratica

$$= \alpha^2\left(1-\frac{a^2}{r}\right)+\beta^2\left(1-\frac{b^2}{r}\right)+$$

$$-\gamma^2\left(1+\frac{c^2}{r}\right)-2\alpha\beta ab+2\alpha\gamma ac+2\beta\gamma bc$$

$$\frac{1}{r}\begin{pmatrix} (b^2-c^2) & -ab & +ac \\ -ab & (a^2-c^2) & +bc \\ +ac & +bc & -(a^2+b^2) \end{pmatrix}$$

Pol. car.:

$$-\lambda^3 - 2c^2\lambda^2 + (-c^4 + b^4 + 2a^2b^2 + a^4)\lambda =$$

$$= -\lambda\left(\lambda^2 + 2c^2\lambda - \underbrace{(a^2+b^2+c^2)}_{>0}\underbrace{(a^2+b^2-c^2)}_{>0}\right)$$

$$\sigma = (1,1)$$

Classificazione proiettiva delle iperq. della retta reale

$r=1$

$\begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$ $\begin{pmatrix} -1 & \\ & 0 \end{pmatrix}$ I_{n_1} $X_0^2 = 0$

$W_1 \begin{cases} X_0 = 0 & \text{un punto (contato 2 volte)} \\ 0 = 0 & \text{lo stesso punto} \end{cases}$

$\Gamma = 2$

$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} -1 & \\ & -1 \end{pmatrix}$ Im 1 $x_0^2 + x_1^2 = 0$

$\emptyset$

$W: \begin{cases} x_0 = 0 \\ x_1 = 0 \end{cases}$ $\emptyset$

$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ Im 1 $x_0^2 - x_1^2 = 0$

$(x_0 + x_1)(x_0 - x_1) = 0$

$W \begin{cases} x_0 = 0 \\ -x_1 = 0 \end{cases}$ $\emptyset$ due punti affinital

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

$\det \neq 0$

$$x' = ax + by + e$$

$$y' \quad \rule{8cm}{0.4pt}$$

$$\frac{x_1'}{x_0'} = a\,\frac{x_1'}{x_0} + b\,\frac{x_2}{x_0} + e$$

$$x_1' = a x_1 + b x_2 + e x_0$$

$$\lambda \begin{pmatrix} x_0' \\ x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ e & a & b \\ f & c & d \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$$

$$\det \neq 0$$

$$\cancel{a_{00}X_0^2} + \cancel{2a_{01}X_0X_1} + \cancel{2a_{02}X_0X_2} +$$

$$+ a_{11}X_1^2 + 2a_{12}X_1X_2 + a_{22}X_2^2 = 0$$

$\frac{1}{X_0^2}$ $X_0 = 0$

$$A = \begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix}$$

$= a_{01}$

ecc.

$= M_{00} = M_0^0$

$A_{00} = A_0^0 = |M_{00}|$

$$a_{00} + 2a_{01}x + 2a_{02}y + a_{11}x^2 + 2a_{12}xy + a_{22}y^2 = 0$$

Classificare le coniche

$$x^2 + 2\gamma xy + 4y^2 - 10 = 0$$

al variare di $\gamma \in \mathbb{R}$

$\forall \gamma$ $\det \neq 0$

$$A = \begin{pmatrix} -10 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & \gamma & 4 \end{pmatrix} \quad M_0 \quad M_{00}$$

di rango 2

$$|A| = -10 \begin{vmatrix} 1 & \gamma \\ \gamma & 4 \end{vmatrix} = -10(4-\gamma^2) = 10(\gamma^2-4) = 10(\gamma+2)(\gamma-2)$$

$\deg \iff \gamma = -2 \vee \gamma = 2$

$$M_0^0 = \begin{pmatrix} 1 & \gamma \\ \gamma & 4 \end{pmatrix}$$

$$A_0^0 = 4 - \gamma^2 \gtrless 0$$

$$-2 \leq \gamma \leq 2$$

γ	$\lvert A \rvert$	rA	M_0^0	coniche
$\gamma < -2$	$\neq 0$	3	$-$	iperboli
$\gamma = -2$	0	2	0	deg. $r=2$
$-2<\gamma<2$	$\neq 0$	3	$+$	ellissi reali
$\gamma = 2$	0	2	0	deg. $r=2$
$\gamma > 2$	$\neq 0$	3	$-$	iperboli

$$\begin{pmatrix} -10 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & \gamma & 4 \end{pmatrix}$$

$M_1 = (-10)$ $\quad |M_1| < 0$ $-$

$M_2 = \begin{pmatrix} -10 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad |M_2| < 0$ $-$

$$\begin{pmatrix} 7 & 5\sqrt{3} & 84 \\ 5\sqrt{3} & -1 & \frac{73}{2} \\ 84 & \frac{73}{2} & \sqrt{931} \end{pmatrix}$$

$+$	$-$
$+$	$+$
$+$	$-$
$+$	$+$
	$-$
def $+$	def $-$

$$\begin{pmatrix} 1 & 5 & 0 \\ 5 & 3 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

(1) $+$

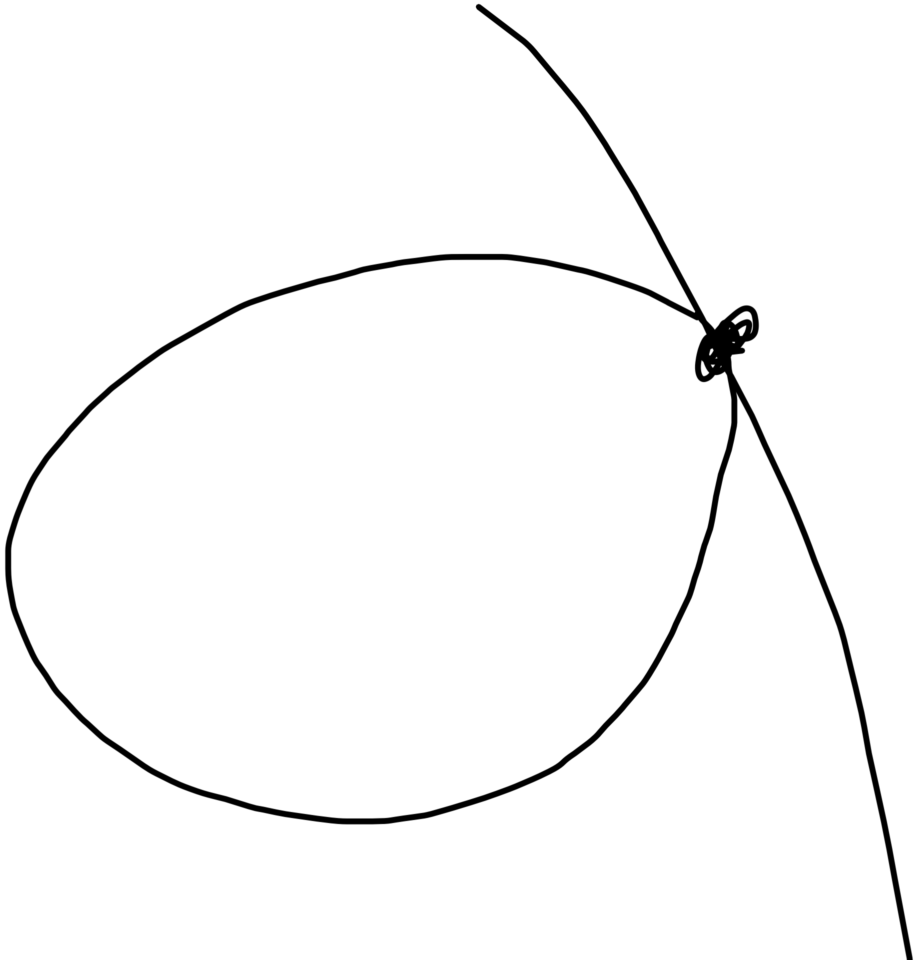
$\begin{pmatrix} 1 & 5 \\ 5 & 3 \end{pmatrix}$ $-$

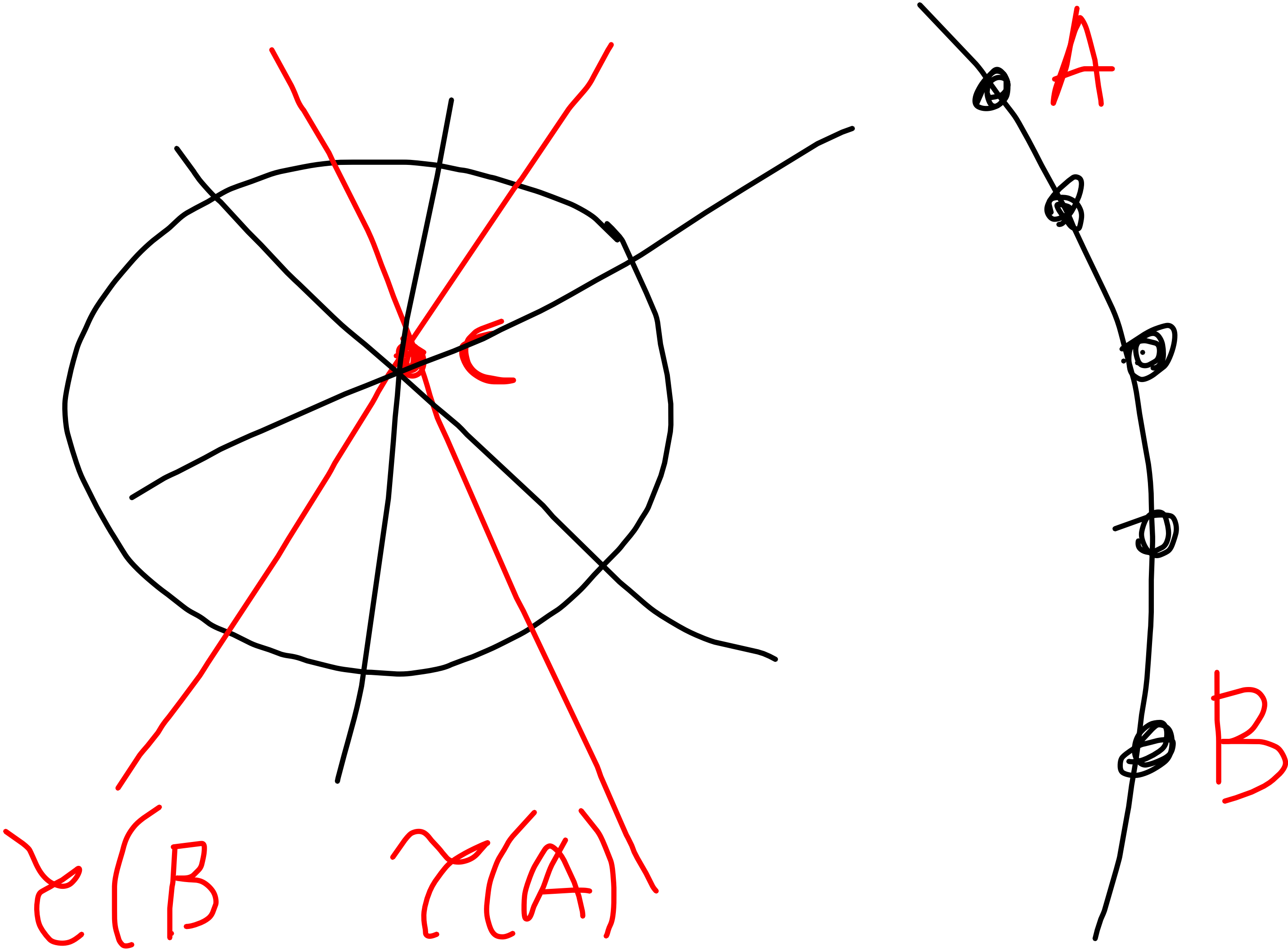
$-$

Classificare le quadriche

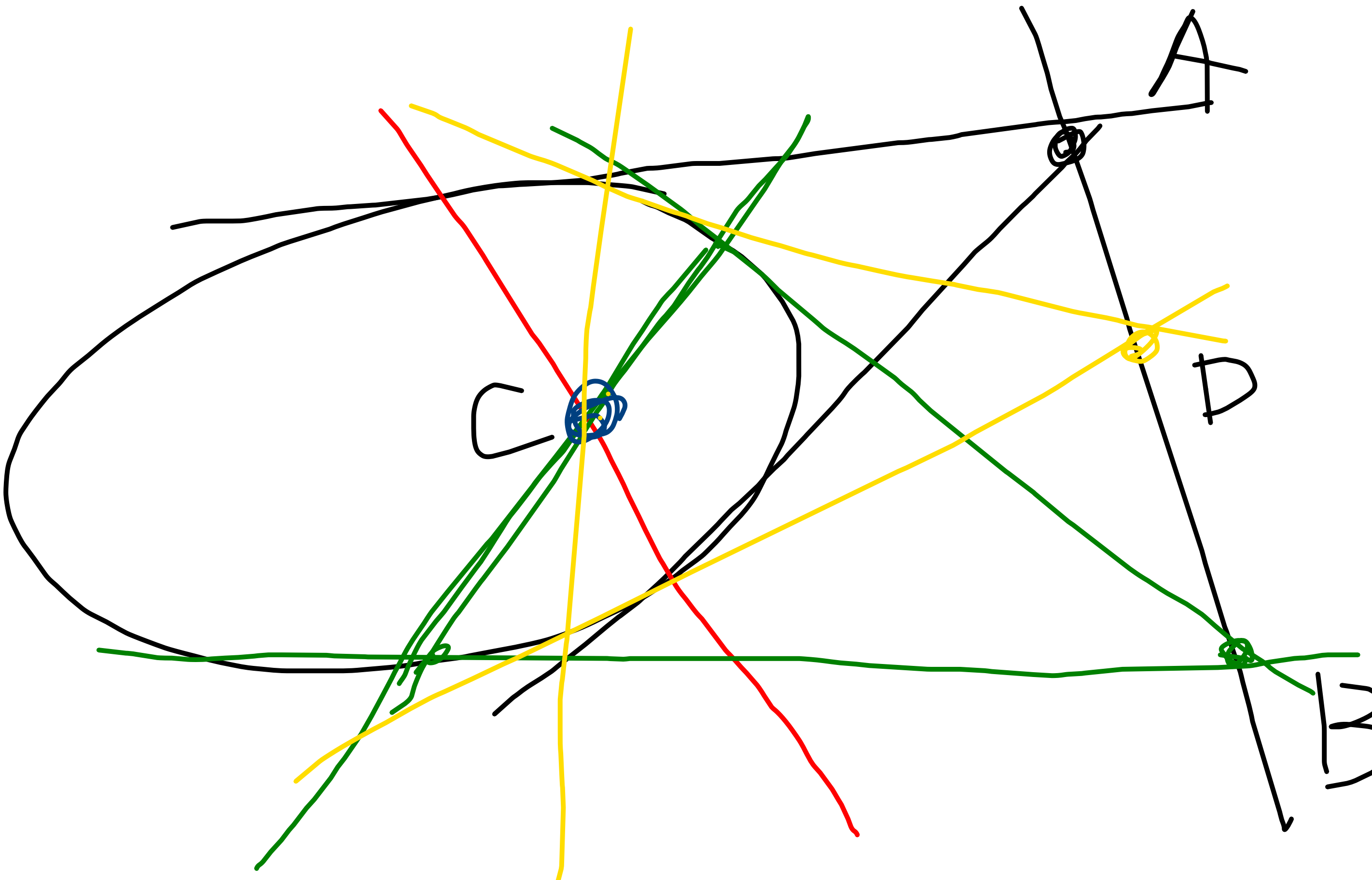
$$(3\lambda-1)x^2+(1-\lambda)y^2+2\lambda xz+2z-4\lambda=0$$

al variare di $\lambda\in\mathbb{R}$





A
B
C
γ(B
γ(A)


A
D
C
B