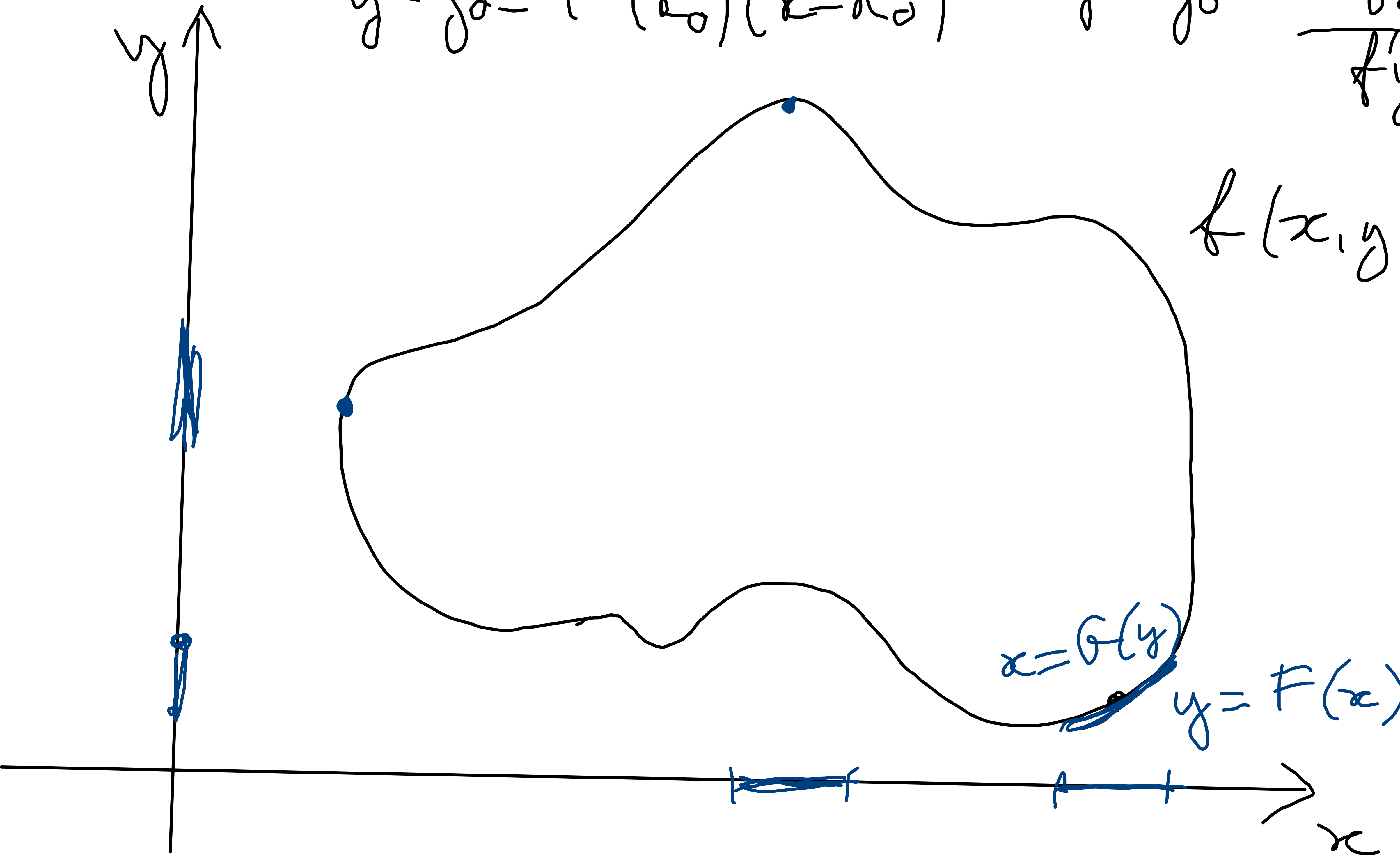
$y - y_0 = F'(x_0)(x - x_0)$ $\qquad$ $y - y_0 = \dfrac{-f'_x(x_0, y_0)}{f'_y(x_0, y_0)} \cdot (x - x_0)$

$f(x, y) = 0$



$$\begin{vmatrix} x & y & 1 \\ f_1(\bar{u}) & f_2(\bar{u}) & 1 \\ f_1'(\bar{u}) & f_2'(\bar{u}) & 0 \end{vmatrix} = \begin{vmatrix} x-f_1(\bar{u}) & y-f_2(\bar{u}) & 0 \\ f_1(\bar{u}) & f_2(\bar{u}) & 1 \\ f_1'(\bar{u}) & f_2'(\bar{u}) & 0 \end{vmatrix} = 0$$

$$x_0 = f_1(\bar{u})$$
$$y_0 = f_2(\bar{u})$$

$$\begin{cases} x = f_1(u) \\ y = f_2(u) \end{cases}$$

$$(x-x_0) f_2'(\bar{u}) - (y-y_0) f_1'(\bar{u}) = 0$$

$$\frac{x-x_0}{f_1'(\bar{u})} = \frac{y-y_0}{f_2'(\bar{u})}$$

$\mathcal{C}: y = x^2 \quad \mathcal{C}': y = -x^3$ luogo $\mathcal{L}$ dei punti d'intersezione delle tangenti a $\mathcal{C}$ e a $\mathcal{C}'$ in punti di uguale ascissa.

$P_\alpha \equiv (\alpha, \alpha^2) \in \mathcal{C} \quad P'_\alpha \equiv (\alpha, -\alpha^3) \in \mathcal{C}'$

$t_\alpha: y - \alpha^2 = 2\alpha(x - \alpha)$ a $\mathcal{C}$ in P_α $\quad y - \alpha^2 = 2\alpha x - 2\alpha^2$

$t': y + \alpha^3 = -3\alpha^2(x - \alpha)$ a $\mathcal{C}'$ in P'_α $\quad y + \alpha^3 = -3\alpha^2 x + 3\alpha^3$

$\mathcal{L}: \begin{cases} t'_\alpha & 2\alpha^3 - 3x\alpha^2 - y = 0 \\ t_\alpha & \alpha^2 - 2x\alpha + y = 0 \end{cases}$

$2\alpha^3 - 3x\alpha^2 - y = 0 \quad | \; \alpha^2 - 2x\alpha + y$

$\mathcal{L}: \dfrac{4y^3 - 3x^2y^2 + 6xy^2 + y^2 - 4x^3y}{4y^2 - 8x^2y + 4x^4} = 0$

$$\begin{vmatrix} 2 & -3x & 0 & -y & 0 \\ 0 & 2 & -3x & 0 & -y \\ 1 & -2x & y & 0 & 0 \\ 0 & 1 & -2x & y & 0 \\ 0 & 0 & 1 & -2x & y \end{vmatrix} = 0$$

$4y^3 - 3x^2y^2 + 6xy^2 + y^2 - 4x^3y = 0$

$$4y^3 - 3x^2y^2 + 6xy^2 + y^2 - 4x^3y = 0$$

$$4y^2 - 8x^2y + 4x^4$$

$$4(y - x^2)^2$$

$$y = x^2$$

$$4x^6 - 3x^6 + 6x^5 + x^4 - 4x^5 =$$

$$= x^4(x^2 + 2x + 1) = x^4(x+1)^2$$

$x = 0$

$\vee$

$x = -1$

$\mathcal{C}: y = x^2$ podaria $\mathcal{P}$ di $\mathcal{C}$ da $A = (3,0)$, cioè luogo dei punti d'intersezione delle tangenti a $\mathcal{C}$ con le rispettive normali condotte da A

$Q = (\alpha, \alpha^2) \in \mathcal{C}$

$t_\alpha: y - \alpha^2 = 2\alpha(x - \alpha)$

$-\alpha^2 + 2x\alpha - y = 0$

$n_\alpha: 1(x-3) + 2\alpha(y-0) = 0$

$x - 3 + 2\alpha y = 0$

$$\mathcal{P}: \begin{cases} -\alpha^2 + 2x\alpha - y = 0 \\ 2y\alpha + x - 3 = 0 \end{cases}$$

$$\begin{cases} -\alpha^2 \ldots \\ \alpha = \dfrac{3-x}{2y} \end{cases}$$

$$-\frac{(3-x)^2}{4y^2} + \frac{6x - 2x^2}{2y} - y = 0$$

$$\frac{-9 - x^2 + 6x + 12xy - 4x^2y - 4y^3}{4y^2} = 0$$

$$\begin{vmatrix} -1 & 2x & -1 \\ 2y & (x-3) & 0 \\ 0 & 2y & (x-2) \end{vmatrix}$$

$$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$

$$(u,v) \longmapsto (u+2v,\ 2u+4v,\ 3u+6v)$$

$$r\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} = 1 = \dim \operatorname{Im} T$$

$$F : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$

$$(u,v) \longmapsto (u^2-6uv,\ \underline{\qquad},\ \quad)$$

$$C : \begin{cases} x = u \\ y = u^2 \\ z = u^3 \end{cases}$$