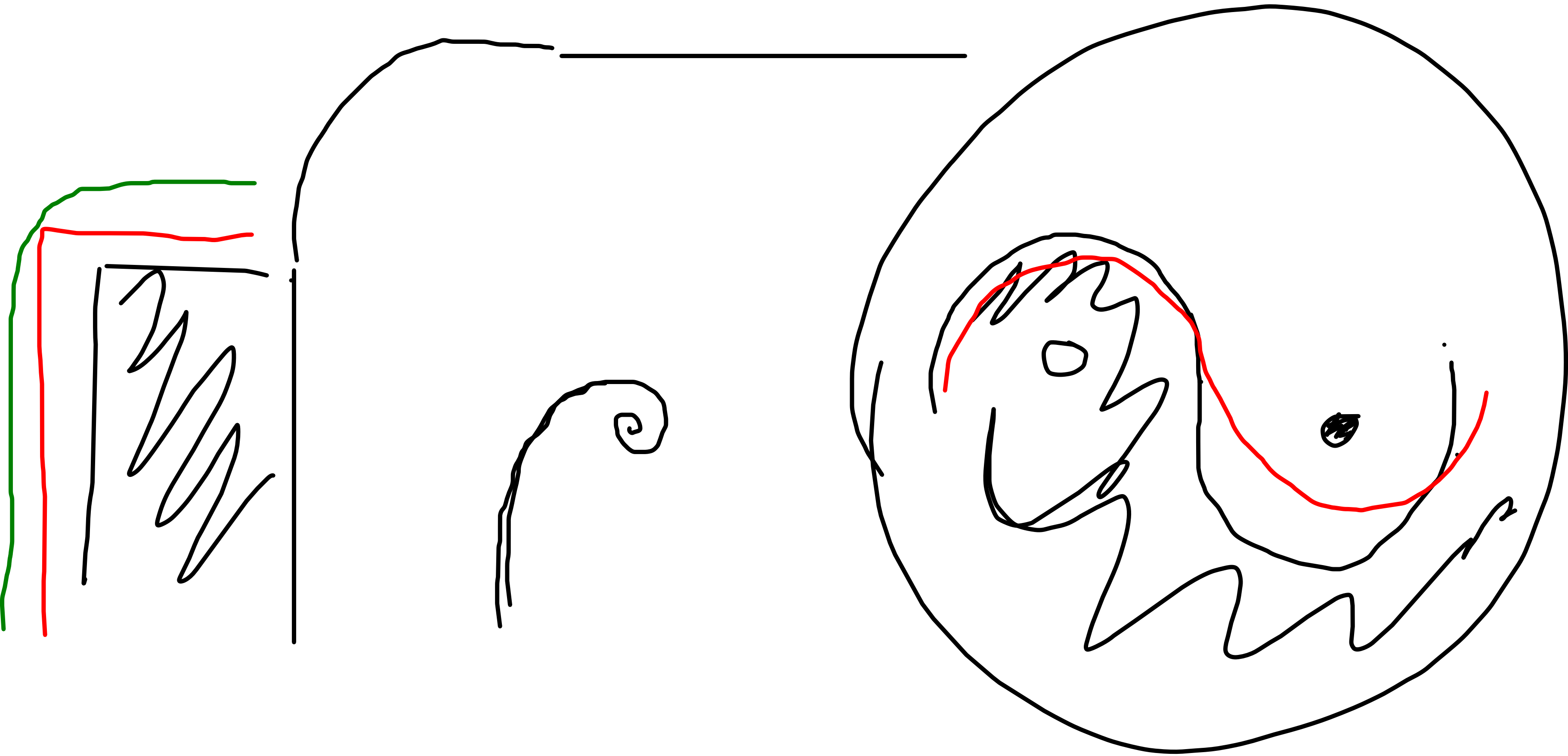

Es. 9 $\quad C: y = 1/x$

Cerchio oscul. in $P \equiv (2, \tfrac{1}{2})$, $Q \equiv (1,1)$

calcolare l'ordine di contatto

$$P_u \equiv \left(u, \tfrac{1}{u}\right) \qquad x^2+y^2+ax+by+c = 0$$

$F(x,y) = 0$

$u=2 \qquad u=1$

$$\Phi(u) = u^2 + \frac{1}{u^2} + au + \frac{b}{u} + c$$

$$\Phi'(u) = 2u - \frac{b}{u^2} - \frac{2}{u^3} + a$$

$$\Phi''(u) = \frac{2b}{u^3} + \frac{6}{u^4} + 2$$

$$\Phi'''(u) = -\frac{6b}{u^4} - \frac{24}{u^5}$$

$$\Phi^{IV}(u) = \frac{24b}{u^5} + \frac{120}{u^6}$$

$$\begin{cases} \Phi(2) = 0 \\ \Phi'(2) = 0 \\ \Phi''(2) = 0 \end{cases}$$

$$x^2 + y^2 - \frac{49}{8}x - \frac{19}{2}y + \frac{51}{4} = 0$$

$\Phi'''(2) \neq 0$ contatto di ord $= 2$

$$\begin{cases} \Phi(1) = 0 \\ \Phi'(1) = 0 \\ \Phi''(1) = 0 \end{cases}$$

$$x^2 + y^2 - 4x - 4y + 6 = 0$$

$\Phi'''(1) = 0$ contatto

$\Phi^{IV}(1) \neq 0$ di ord $= 3$

$(x_0, y_0) \in C \mid f(x_0, y_0) = 0$

$r: y - y_0 = k(x - x_0)$ $\quad C \cap r$

$\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0, y_0)$

$$f(x, y) = f(x_0, y_0) + \left((x-x_0)\frac{\partial}{x} + (y-y_0)\frac{\partial}{y}\right) f(x_0, y_0) +$$

$$+\frac{1}{2}\left((x-x_0)^2\frac{\partial^2}{\partial x^2} + 2(x-x_0)(y-y_0)\frac{\partial^2}{\partial x \partial y} + (y-y_0)^2\frac{\partial^2}{\partial y^2}\right) f(x_0, y_0) +$$

$+$

$$\cdots + \frac{1}{(s-1)!}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^{(s-1)} f(x_0, y_0) +$$

$$+ \frac{1}{s!}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^{s} f(x_0, y_0) +$$

$$+ \cdots$$

$e \quad n \quad q$

$$\begin{aligned}0 = \underbrace{\begin{aligned}&(x-x_0)\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f(x_0, y_0) + \\ &+ \frac{1}{2}(x-x_0)^2\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(x_0, y_0) + \\ &+ \frac{1}{(5-1)!}(x-x_0)^{(5-1)}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{(5-1)} f(x_0, y_0) +\end{aligned}}_{= 0 \;\forall k} \\ + \frac{1}{5!}(x-x_0)^5\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^5 f(x_0, y_0) + \text{—}\end{aligned}$$

11/9/'09 Es 4a

$\mathcal{L}$ $f(x,y) = x^2 y - x y^2 - x + y = 0$

Trovare punti multipli reali e tangenti in essi

$f_x = 2xy - y^2 - 1$

$f_y = x^2 - 2xy + 1$

$f_{xx} = 2y \quad f_{xy} = 2x - 2y \quad f_{yy} = -2x$

$$\begin{cases} f = 0 \\ f_x = 0 \\ f_y = 0 \end{cases} \qquad \begin{cases} f = 0 \\ 2xy - y^2 - 1 = 0 \\ x^2 - 2xy + 1 = 0 \end{cases} \qquad \begin{cases} f = 0 \\ 2xy - y^2 - 1 = 0 \\ x^2 = y^2 \end{cases}$$

$$\begin{cases} f = 0 \\ 2xy - y^2 - 1 = 0 \\ x = \pm y \end{cases} \quad \begin{matrix} + \ 1 \\ - \ 2 \end{matrix}$$

$$1 \qquad \begin{cases} f = 0 \\ y^2 = 1 \\ x = y \end{cases} \qquad \begin{cases} f = 0 \\ y = \pm 1 \\ x = \pm 1 \end{cases}$$

$(1, 1)$

$(-1, -1)$

$$-\ \begin{cases} -1 + 1 + 1 - 1 = 0 \\ \end{cases}$$

$$x^2y - xy^2 - x + y = 0$$

$$\begin{cases} y = \pm 1 \\ x = \pm 1 \end{cases} \quad + \begin{cases} 1 - 1 - 1 + 1 = \\ = \end{cases}$$

② $\begin{cases} f = 0 \\ 2xy - y^2 - 1 = 0 \\ x = -y \end{cases}$ $\begin{cases} f = 0 \\ -2y^2 - y^2 - 1 = 0 \\ x = -y \end{cases}$ no real

in $(1, 1)$

$f_{xx} = 2$

$f_{xy} = 0$

$f_{yy} = -2$

$2 + 0k - 2k^2 = 0$

$k = \pm 1$

$(y-1) = (x-1)$

$(y-1) = -(x-1)$