$x^3 + y^3 - 3xy = 0$

$F =$

$F_{xx} = 6x$ in 0

$F_{xy} = -3$ $\quad -3$

$F_{yy} = 6y$ $\quad 0$

$F_x = 3x^2 - 3y$

$F_y = 3y^2 - 3x$

$y = kx$

$\beta y = \alpha x$

$$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(0,0) = 0$$

$0 - 6k + 0k^2 = 0$

$k = 0$

$$\left(\alpha\frac{\partial}{\partial x} + \beta\frac{\partial}{\partial y}\right)^2 f(0,0) = 0$$

$0\alpha^2 - 6\alpha\beta + 0\beta^2 = 0$

$-6\alpha\beta = 0$ $\quad \alpha = 0 \ \beta \neq 0$; $\quad \beta = 0 \ \alpha \neq 0$

Es. 11 $\quad \mathcal{C}: 4x^2y - y^3 + 6xy - 8y + 1 = 0 \qquad x = \frac{X_1}{X_0} \quad y = \frac{X_2}{X_0}$

$= 4\frac{X_1^2}{X_0^2}\frac{X_2}{X_0} \qquad = 6\frac{X_1}{X_0}\frac{X_2}{X_0}$

Trovare asintoti $\quad 4X_1^2X_2 - X_2^3 + 6X_1X_2X_0 - 8X_2X_0^2 + X_0^3$

$$F = 4x^2y - y^3 + 6\cancel{xyt} - \cancel{8yt^2} + \cancel{t^3} = 0 \quad \begin{cases} y(4x^2 - y^2) = 0 \\ t = 0 \end{cases}$$

$t = 0$

$$\begin{cases} y = 0 \\ t = 0 \end{cases} \vee \begin{cases} y = \pm 2x \\ t = 0 \end{cases}$$

$A_\infty = (0,1,0) \quad B_\infty = (0,1,2) \quad C_\infty = (0,1,-2)$

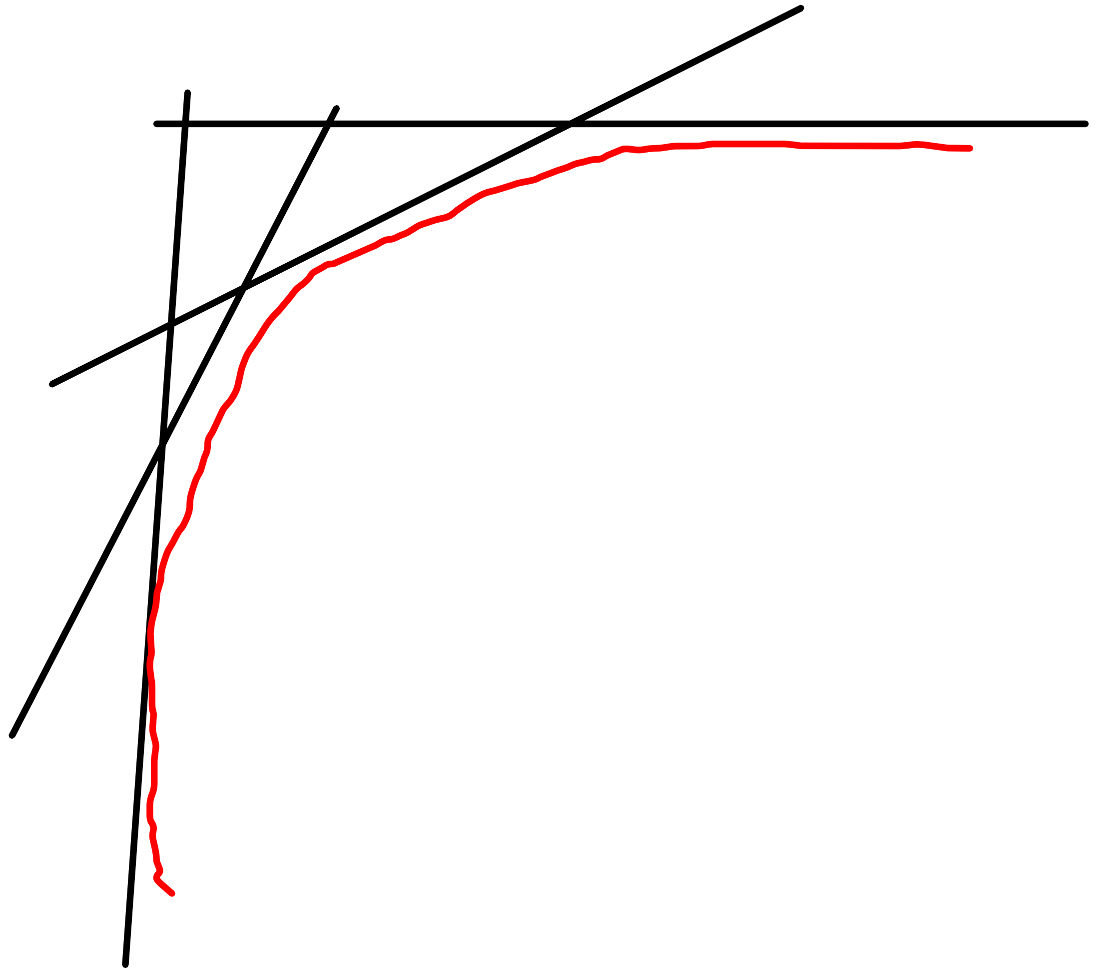
	A_∞	B_∞	C_∞
$F_x = 8xy + 6yt$	0	16	
$F_y = 4x^2 - 3y^2 + 6xt - 8t^2$	4	-8	
$F_t = 6xy - 16yt + 3t^2$	0	12	

$t_{A_\infty}: 0t + 4x + 0y = 0$

$x = 0$

$t_{B_\infty}: 16t - 8x + 12y = 0$

$16 - 8x + 12y = 0$



Trovare la curva inviluppo del sistema di rette

$(1-3\alpha^2)x + y + 2\alpha^3 = 0$ $\qquad F = 2\alpha^3 - 3x\alpha^2 + x + y$

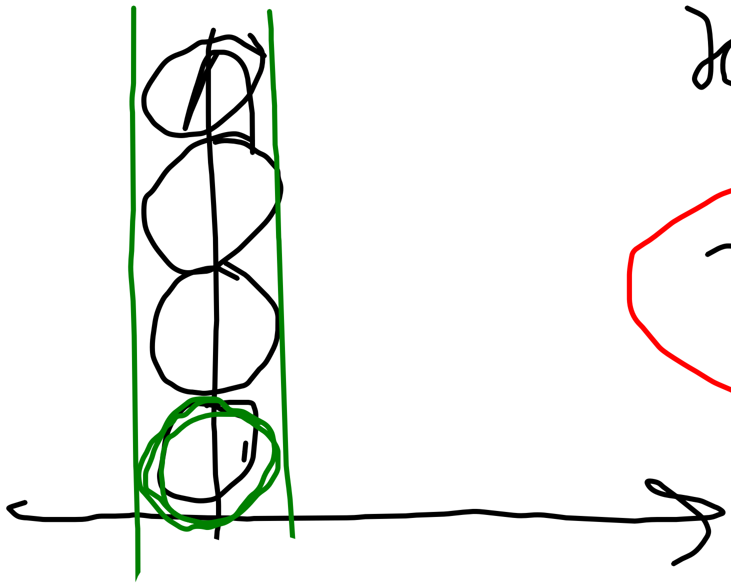
$F_\alpha = 6\alpha^2 - 6x\alpha = 0$

$$3F - \alpha F_\alpha = 6\alpha^3 - 9x\alpha^2 + 3x + 3y +$$
$$-6\alpha^3 + 6x\alpha^2$$

// $-3x\alpha^2 + 3x + 3y = 0$

$$\begin{cases} \alpha^2 - x\alpha = 0 \\ x\alpha^2 - x - y = 0 \end{cases}$$

$$\begin{vmatrix} 1 & -x & 0 & 0 \\ 0 & 1 & -x & 0 \\ x & 0 & -x-y & 0 \\ 0 & x & 0 & -x-y \end{vmatrix} = (y+x)(y-x^3+x) = 0$$



$$x^2+(y-u)^2-1=0$$

$F=$

$$x^2+y^2-2uy+u^2-1=0$$

$$F_u=-2y+2u$$

$$\begin{cases} F=0 \\ F_u=0 \end{cases} \qquad \begin{cases} x^2 \text{———} \\ u=y \end{cases}$$

$$x^2+\cancel{y^2}-2\cancel{y^2}+\cancel{y^2}-1=0$$

$$x^2-1=0$$

$$x^2+(y-u^2)^2-1=0$$

F

$$x^2+y^2-2\underset{v}{u^2}y+\underset{v^2}{u^4}-1=0$$

$$F_u=-4uy+4u^3=$$

$$=4u(-y+u^2)$$

$$\begin{cases} F=0 \\ F_u=0 \end{cases} \quad ① \; u=0, \quad ② \; y=\underset{v}{u^2}$$

① ∪ ② $\quad (x^2+y^2-1)(x^2-1)=0$

10/9/'10 Es 4b triedro fond. in A corr. a $t=0$

di $\mathcal{L}$: $\begin{cases} x = t^2-1 \\ y = t^2-t \\ z = -2t^2 \end{cases}$ $\quad x' = 2t \quad y' = 2t-1 \quad z' = -4t \quad x'' = 2 \quad y'' = 2 \quad z'' = -4$

$t=0 \quad A \equiv (-1,0,0)$

tang.: $\cancel{\dfrac{x+1}{0} = \dfrac{y}{-1} = \dfrac{z}{0}}$ $\quad \boxed{t: \begin{cases} x+1=0 \\ z=0 \end{cases}}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $(l,m,n) \sim \left(\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \right) \sim (0,1,0)$

p. osc. π_o: $\begin{vmatrix} x+1 & y & z \\ 0 & -1 & 0 \\ 2 & 2 & -4 \end{vmatrix} = 0 \quad -(-4)(x+1) - 2z = 0$

$\boxed{\pi_o: 4x - 2z + 4 = 0 \\ 2x - z + 2 = 0}$

p. norm.: π_n: $0(x+1) + 1(y-0) + 0(z-0) = 0$ $\quad \boxed{\pi_n: y=0}$

norm. princ. $\boxed{n: \begin{cases} 2x - z + 2 = 0 \\ y = 0 \end{cases}}$

binorm.: b: $\cancel{\dfrac{x+1}{2} = \dfrac{y}{0} = \dfrac{z}{-1}}$ $\quad \boxed{b: \begin{cases} \dfrac{x+1}{2} = \dfrac{z}{-1} \\ y = 0 \end{cases}}$

$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ $(\bar{l}, \bar{m}, \bar{n})_n (1, 0, 2)$

p. rettif. π_r: $1(x+1) + 0(y-0) + 2(z-0) = 0$ $\quad \boxed{\pi_r: x + 2z + 1 = 0}$

$$x^2+y^2+z^2+ax+by+cz+d=0$$

$$\begin{cases} x = f(u) \\ y = \varphi(u) \\ z = \psi(u) \end{cases}$$

$$P \longleftrightarrow u = \bar{u}$$

$$\cancel{F} =$$

$$\Phi(u) = F\big(f(u), \varphi(u), \psi(u)\big)$$

$$\begin{cases} \Phi(\bar{u}) = 0 \\ \Phi'(\bar{u}) = 0 \\ \Phi''(\bar{u}) = 0 \end{cases}$$

$\vec{t}$ versore tang. $\vec{n}$ versore norm. princ.,
$\vec{b}$ versore binormale.
$\varkappa$ flessione τ torsione in P

$$\begin{pmatrix} \vec{t}\,' \\ \vec{n}\,' \\ \vec{b}\,' \end{pmatrix} = \begin{pmatrix} 0 & \varkappa & 0 \\ -\varkappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix} \begin{pmatrix} \vec{t} \\ \vec{n} \\ \vec{b} \end{pmatrix}$$

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spf Σ punto $P \in \Sigma$ Π tang. a Σ in P
gener. retta r tang. in P a Σ

$\mathcal{C} = \Sigma \cap \Pi$ quante intersez raccolte in P fra r e $\mathcal{C}$? 2

12/12/'08 Es 3b

Σ: $3x^3 - 3x^2y - xz + xy^2 + 2x + yz - y^3 - zy = 0$

$F =$

Verif. che $P \equiv (0,0,2) \in \Sigma$, con c tang. in P

$F_x = 9x^2 - 6xy - z + y^2 + z$ $\quad 0$

$F_y = -3x^2 + 2xy + z - 3y^2 - z$ $\quad 0$

$F_z = -x + y$ $\quad 0$

P

$F_{xx} = 18x - 6y$ $\quad 0$

$F_{xy} = -6x + 2y$ $\quad 0$

$F_{xz} = -1$ $\quad -1$

$F_{yy} = 2x - 6y$ $\quad 0$

$F_{yz} = 1$ $\quad 1$

$F_{zz} = 0$ $\quad 0$

$F_{xx} = 18x - 6y$ $\quad 0$

$F_{xy} = -6x + 2y$ $\quad 0$

$F_{xz} = -1$ $\quad -1$

$F_{yy} = 2x - 6y$ $\quad 0$

$F_{yz} = 1$ $\quad 1$

$F_{zz} = 0$ $\quad 0$

$$\left((x-0)\frac{\partial}{\partial x} + (y-0)\frac{\partial}{\partial y} + (z-2)\frac{\partial}{\partial z}\right)^2 f(0,0,2) = 0$$

$$0x^2 + 2\cdot 0\, x\, y + 2(-1)\, x(z-2) + 0y^2 + 2\cdot 1\cdot y\cdot(z-2) + 0(z-2)^2 = 0$$

$$-2x(z-2) + 2y(z-2) = 0$$

$$(z-2)(y-x) = 0$$