11/9/09 Es 4a $\quad C \quad F(x,y) = x^2y - xy^2 - x + y = 0$

Trovare pti multipli reali e le tang. inerenti dessi

$F_x = 2xy - y^2 - 1 \qquad F_y = x^2 - 2xy + 1$

$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \end{cases}$

$\begin{cases} F = 0 \\ 2xy - y^2 - 1 = 0 \\ x^2 - 2xy + 1 = 0 \end{cases} \quad \begin{cases} F = 0 \\ 2xy - 1 = y^2 \\ 2xy - 1 = x^2 \end{cases} \quad \begin{cases} F = 0 \\ 2xy - y^2 - 1 = 0 \\ x^2 = y^2 \end{cases}$

$\begin{cases} F = 0 \\ 2xy - y^2 - 1 = 0 \\ x^2 = y^2 \end{cases}$ $\quad y = x \quad \begin{cases} F = 0 \\ 2x^2 - x^2 - 1 = 0 \\ y = x \end{cases} \quad \begin{cases} F = 0 \\ x^2 = 1 \\ y = x \end{cases} \quad \begin{cases} F = 0 \\ x = \pm 1 \\ y = x \end{cases}$

$(1,1) \quad \begin{cases} 0 = 0 \\ x = 1 \\ y = 1 \end{cases} \qquad (-1,-1) \quad \begin{cases} 0 = 0 \\ x = -1 \\ y = -1 \end{cases}$

$y = -x \quad \begin{cases} F = 0 \\ 2xy - y^2 - 1 = 0 \\ y = -x \end{cases} \quad \begin{cases} F = 0 \\ -2x^2 - x^2 - 1 = 0 \\ y = -x \end{cases} \quad \begin{cases} F = 0 \\ 3x^2 = -1 \\ y = -x \end{cases}$

$F_{xx} = 2y \qquad F_{xy} = 2x - 2y \qquad F_{yy} = -2x$

in $(1,1)$: $\quad 2 \qquad 0 \qquad -2$

in $(-1,-1)$: $\quad -2 \qquad 0 \qquad +2$

$2 \quad -2k^2=0 \quad k=\pm 1$ tangenti in $(1,1)$: $(y-1)=1(x-1)$

$(y-1)=-1(x-1)$

$2(x-1)^2-2(y-1)^2=0$

$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)^2 F(x_0,y_0)=0$$

$$\left((x-1)^2\,2+(y-1)^2(-2)\right)=0$$

$(x-1)^2-(y-1)^2=0 \quad (x-1+y-1)(x-1-y+1)=0$

$$\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^2 F(x_0,y_0)=0$$

$$\left(F_{xx}^{(1,1)}+2F_{xy}^{(1,1)}k+F_{yy}^{(1,1)}k^2\right)=0$$

in $(-1,-1)$

$-2+2k^2=0 \quad k=\pm 1$

tang.: $(y+1)=1(x+1)$

$(y+1)=-1(x+1)$

$\left((x+1)^2(-2)+(y+1)^2\,2\right)=0$

$+(x+1)^2-(y+1)^2=0$

$(x+1+y+1)(x+1-y-1)=0$

22/7/'09 Es la pti multipli reali tangenti in una di essi di:

$C: x^4 - y^4 + xy = 0$

$F(x,y) =$

$F_x = 4x^3 + y \qquad F_y = -4y^3 + x$

$F_{xx} = 12x^2 \qquad F_{xy} = 1 \qquad F_{yy} = -12y^2$

in $(0,0)$: $0 \qquad 1 \qquad 0$

$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \end{cases}$

$\begin{cases} F = 0 \\ y = -4x^3 \\ x = 4y^3 \end{cases} \qquad \begin{cases} F = 0 \\ x = 4y^3 \\ y = -4(4y^3)^3 \end{cases} \qquad y = -256\,y^9 \qquad \begin{cases} F = 0 \\ x = 4y^3 \\ y(256y^8 + 1) = 0 \end{cases}$

$\begin{cases} F = 0 \\ x = 4y^3 \\ y = 0 \end{cases} \qquad \begin{cases} 0 = 0 \\ x = 0 \\ y = 0 \end{cases} \qquad (0,0) \qquad xy = 0$

unione delle tangenti in O

$k = \frac{b}{a}$

$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 F(0,0) = 0 \qquad 0 + 2 \cdot 1 \cdot k + 0k^2 = 0 \qquad k = 0 \qquad y = kx$

$a^2\left(0 + 2\frac{b}{a} + 0\left(\frac{b}{a}\right)^2\right) = 0 \qquad 2ab = 0 \qquad by = ax$

$$C : x^4 - y^4 + x^2 y - x y^2 = 0$$

$F_x = 4x^3 + 2xy - y^2$ $\quad F_y = -4y^3 + x^2 - 2xy$ $\quad \left.\begin{array}{l} F = 0 \\ F_x = 0 \\ F_y = 0 \end{array}\right\}$ $\quad (0,0)$

$F_{xx} = 12x^2 + 2y$ $\quad F_{xy} = 2x - 2y$ $\quad F_{yy} = -12y^2 - 2x$

in 0: $\quad 0 \quad 0 \quad 0$

$k = \frac{b}{a}$ $\quad y = kx$

$y = \frac{b}{a}x$

$ay = bx$

$F_{xxx} = 24x$ $\quad F_{xxy} = 2$ $\quad F_{xyy} = -2$ $\quad F_{yyy} = -24y$

in 0: $\quad 0 \quad 2 \quad -2 \quad 0$

$y = 0$ $\quad k = 0$

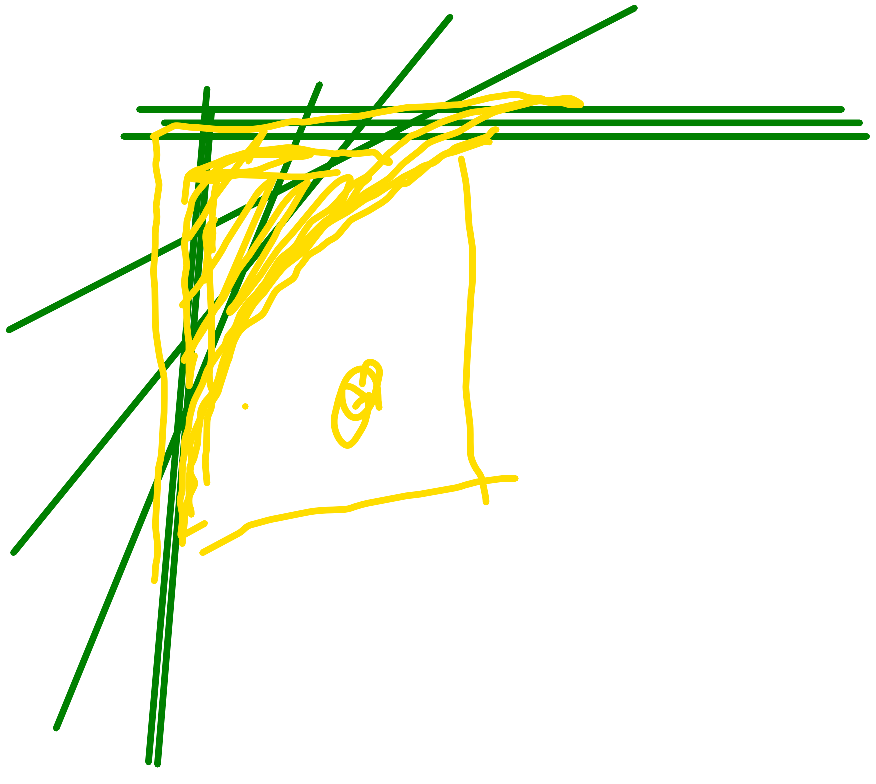
$0 + 2k - 2k^2 + 0k^3 = 0$ $\quad k^2 - k = 0$ $\quad k(k-1) = 0$

$x = 0$ $\quad y = x$ $\quad k = 1$

$$a^3\left(0 + 2\frac{b}{a} - 2\frac{b^2}{a^2} + 0\frac{b^3}{a^3}\right) = 0$$

$0 + 2ba^2 - 2b^2 a = 0$ $\quad 2ba(a-b) = 0$

- $b = 0$ $\quad y = 0$
- $a = 0$ $\quad x = 0$
- $a = b$ $\quad y = x$



9/11/'09 Es 4 Inviluppo (eventualmente unito alle rette stazionarie) della famiglia di rette

$$F(x,y,\alpha) = 3\alpha^2 x + (3\alpha^2 - 1)y - 2\alpha^3 = 0$$

$$F_\alpha = 6\alpha x + 6\alpha y - 6\alpha^2 = -6\alpha^2 + 6\alpha(x+y)$$

$$F = -2\alpha^3 + 3(x+y)\alpha^2 - y$$

$$\mathcal{E} : \begin{cases} F = 0 \\ F_\alpha = 0 \end{cases} \qquad \begin{cases} -2\alpha^3 + 3(x+y)\alpha^2 - y = 0 & F = \\ -6\alpha^2 + 6\alpha(x+y) = 0 & F' = \end{cases}$$

$$\alpha^2 - \alpha(x+y) = 0 \qquad \begin{cases} F' = 0 \\ F + 2\alpha F' = 0 \end{cases}$$

$$F + 2\alpha F' = -2\alpha^3 + 3(x+y)\alpha^2 - y + 2\alpha^3 - 2\alpha^2(x+y)$$

$$// \quad (x+y)\alpha^2 - y$$

$$\begin{cases} \alpha^2 - \alpha(x+y) = 0 \\ (x+y)\alpha^2 - y = 0 \end{cases} \qquad \begin{cases} \alpha(\alpha - (x+y)) = 0 \\ (x+y)\alpha^2 - y = 0 \end{cases}$$

$$\begin{cases} \alpha = 0 \\ \_\_\_ \end{cases} \qquad \begin{cases} \alpha = x+y \\ \_\_\_ \end{cases}$$

$$\begin{vmatrix} 1 & -(x+y) & 0 & 0 \\ 0 & 1 & -(x+y) & 0 \\ (x+y) & 0 & -y & 0 \\ 0 & (x+y) & 0 & -y \end{vmatrix} = 0$$

$$-y^4 - 3xy^3 - 3x^2y^2 + y^2 - x^3 y = 0$$

$$y(-y^3 - 3xy^2 - 3x^2 y + y - x^3) = 0$$